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WARPING STRESSES AND DEFLECTIONS IN CONCRETE PAVEMENTS

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by

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Joint Highway Research Project

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Technical Paper

WARPING STRESSES AND DEFLECTIONS IN CONCRETE PAVEMENTS

TO: K. B. Woods, Director
Joint Highway Research Project

January 29, 1959

FROM: H. L. Michael, Assistant Director
Joint Highway Research Project

File: 9-7-2
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Attached is a paper entitled, "Warping Stresses and Deflections in Concrete Pavements." It was prepared by Professors M. E. Harr and G. A. Leonards of our staff.

The paper was presented at the 38th Annual Meeting of the Highway Research Board in Washington, D. C., in January, 1959.

It is presented for the record to the Board.

Respectfully submitted,

H. L. Michael

H. L. Michael, Secretary

HLM:ac

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Technical Paper

WARPING STRESSES AND DEFLECTIONS IN CONCRETE PAVEMENTS

by

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and
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Joint Highway Research Project
Project No: C-36-63B
File No: 9-7-2

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Lafayette, Indiana

January 29, 1959

INTRODUCTION

General Considerations

According to the Bureau of Public Roads Tabulations, there are approximately 135,000 miles of Portland cement concrete roads in use in the United States, as of the end of 1956. For the past few years, about 2,500 miles of such pavements have been constructed each year. Farrel and Paterick (13) state that expenditures for all types of surfacing on primary and secondary roads have comprised about 40 percent of the construction funds for highways as compared with 25 percent, each, for grading and structures. Accordingly, expenditures for surfacing roads represent a large fraction of the nation's highway investment.

According to Farrell and Paterick (13), the average service life to zero salvage for all high type pavements increased slightly from 27 years in the period 1914-1919 to 33 years during 1930-1939, then decreased to 25 years between 1950-1952; on the other hand, the average service life of highway structures has increased steadily from 40 years to 52 years in the same period. A more recent study by Gronberg and Blosser (22) shows that the average expected service life of Portland cement concrete pavements, which includes replacement resulting from all causes, has averaged about 25 years during the period 1916-1952. In addition, a number of concrete pavements are resurfaced each year in an attempt to halt deterioration and restore their life, although such attempts have not been overly effective from the standpoint of the development of reflection cracking (5, 48).

Although the development of cracks in concrete pavements are not necessarily indicative of impending failure, cracks in existing roads do reflect deficiencies in design and construction practices. The occurrence of these

[illegible] cracks may be injurious initially not primarily from the standpoint of driving comfort, as pointed out by Cassell and Fiske [10], but rather from the danger of water penetration with subsequent [illegible] of subgrade support and increased pavement deterioration. Personal observations on pavements constructed in the 1920's reported the occurrence of many types of cracks, including corner or diagonal cracks, longitudinal cracks, and transverse cracks [9, [illegible], 65]. The introduction of a center joint around 1925 has almost completely eliminated longitudinal cracks near the centerline of the pavement. Historical development of design trends in Portland cement concrete pavements are presented in references [14 and 76]. Application of soil mechanics principles to pavement design has largely eliminated poor subgrade support as a primary cause of concrete pavement failures [8, 11, 26, 59, 71, 72, 73, 74]. Frost susceptible soils have been isolated and generally effective remedial measures have become common practice [11, 26, 36, 71, 72, 74, 75]. The use of granular filter layers has substantially eliminated failures due to pavement pumping [46, 74, 75, 77]. According to Friberg [16], "Poor subgrades cannot be disregarded as a continuing deteriorating influence. But, with the exception of localized frost heave or unusual disturbances of soil equilibrium, subgrade instability appears to be serious exclusively at joints and cracks. Even subgrade pumping, a major sign of deterioration of the pavement, has not been a consequence away from joints (edges) and cracks." Deterioration of unsound concrete is still somewhat of a problem, but the factors involved are now comparatively well known and specifications to combat their deleterious effects are widespread [3, 37, 73]. As stated by Van Breemen [7], "Almost all of the failures that have occurred with concrete pavements in New Jersey have been of a structural nature, as differentiated from failures attributable to weakness or disintegration of the concrete itself."

Recent performance surveys show a predominance of transverse cracks in pavements (1, 10, 25, 26, 58, 70). For example, on 900 miles of Louisiana pavement, Friberg (16) reports 86 percent of all cracks were transverse; corner cracks averaged less than one per mile. On the Maryland Test Road (26), sections 1, 2, and 3, 98 percent of the cracks were transverse cracks. In 1948, Allen and Jackson reporting on a survey of the concrete pavement on the German autobahn (29), stated, "Transverse cracking was common and was not confined to any particular part of the system. There seems to be no relation between transverse cracking, terrain or soil conditions." Similar observations for continually reinforced concrete pavements has also been reported (10).

It has long been recognized that critical stresses in sound concrete pavements are due to the combined effects of pavement warping and superimposed traffic loads (15, 16, 26, 29, 58, 67). The procedures that have evolved to account for these effects will be discussed later; however, it is the writers' opinion that the critical conditions have not been properly evaluated, for less than optimum performance has been obtained from concrete pavements and the remedial measures designed for them (5, 10, 13, 18, 36, 48). Recent experimental studies and observations (12, 17, 27, 28), have clearly demonstrated that as a consequence of warping the assumption of complete pavement contact with its support may be seriously in error. Thus, improved design concepts are predicated on the development of analytical procedures that allow for the possibility that warping may result in the pavement being only partially supported when it receives traffic loads. The presentation of such a theory is a primary purpose of this paper.

Previous Studies of Warping Effects

As early as 1910, the Office of Public Roads (19) conducted field and laboratory tests to study the effects of expansion and contraction of concrete pavements. In 1922, tests conducted on the Bates Experimental Road (41) led to

the discovery that temperature differences between pavement surfaces result in considerable warping of the slab. In 1925, Hatt (23) reported warping could be induced by corresponding moisture differences between the slab surfaces. Beginning in 1930 and continuing to their final report in 1943, the Public Roads Administration conducted extensive investigations on the structural action of concrete pavement slabs. In summarizing these data, Teller and Sutherland stated (58):

> "The data that have just been presented clearly indicate that the stresses arising from restrained temperature warping equal in importance those caused by the heaviest wheel loads. The stresses from this cause are actually large enough to cause failure in concrete of low flexural strength, and since the direction of the stresses is such that they become added to the critical stresses caused by wheel loads, there is little doubt but that warping stress is primarily responsible for much of the cracking in concrete pavements."

In 1944, the California Division of Highways began intensive field and laboratory studies to determine the cause of distress at the joints of concrete pavements. Reporting the results of this study in 1951, Hveem (27) commented as follows: "Engineers have paid much attention to subgrade support for concrete pavements at or near the planned joints in the pavement, but if it is recognized that for a considerable portion of time rigid pavements do not rest on the subgrade for several feet either side of the joint, it seems pertinent to ask whether Westergaard's K is a significant index for the design of such pavements." More recently, Hveem (Figure 45 of reference 28) presented a view through a core hole in a pavement slab which shows approximately 1/4 inch clearance between the slab bottom and the subbase. The slab also shows a crack in this area. In reference to this illustration, Hveem states, "The existence of this void space

'sets the stage' for pumping. There is reason to believe that the so-called pumping action would never start if the slab were firmly in contact with the subgrade at all times."

A survey of the performance of industrial floors on ground made by the Missouri River Division, Corps of Engineers, in 1955 (12) noted voids under slabs as much as 3/8" deep, extending 2 to 5 feet away from the joint. It was observed that the high points occurred at the corners of these slabs. As the slabs were inside buildings and not subject to large changes in temperature, it was reasonsd that the warping was primarily the result of moisture differences between slab surfaces. Geldmacher, et al, (17), in 1957, determined experimentally that the load bearing characteristics of a highway pavement varied diurnally with environmental changes. In some cases, deflections due to warping exceeded those induced by the superimposed traffic loads.

From the evidence cited, it may be concluded that temperature and moisture gradients that develop in concrete pavements can, under prevalent field conditions, cause sufficient warping of the slab to result in only partial support from its base. Analytical tools developed to deal with this phenomenon will now be outlined.

Available Theories

To account for the loss of support at the corners of pavements resulting from such exigencies as weak subgrades, and temperature and moisture differentials, Goldbeck (20) in 1919, and Older (40) in 1921, independently, derived an equation for the required thickness of a concrete pavement assuming that the corner was entirely unsupported. The concepts used in the development of this equation, which is known as the "corner formula", were destined to serve as a basis for most of the formulae in current use.

The first correctly reasoned theoretical analysis of the warping action of concrete pavement slabs was contributed by Westergaard (66) in 1926. In 1927 Westergaard (6[illegible]) [illegible] the stresses and deflections induced in the slab [illegible] "the generality and simplicity of Westergaard's [illegible] led to their early adoption and widespread use in design [illegible] the effect of adjacent loads (56), [illegible] (60) [illegible] (55) have since been added. [illegible] (57). In 1930 [illegible] ed that the temperature differential that [illegible] large as they are. After [illegible] ed in the Arlington investigation (30) [illegible] this analysis might not be applicable for [illegible] of these [illegible] attitudes."

In 1940 [illegible] (61) modified Westergaard's approach by assuming a simple harmonic temperature variation on the top surface of the slab. Applying the law of heat flow, Thomlinson arrives at a [illegible] temperature gradient through the depth of the slab, which corresponds fairly closely with the cyclic daily observations of Teller and Sutherland (18), and [illegible] (34). On the basis of their studies, Teller and Sutherland concluded that "Although the curved gradient represents that more usual distribution of temperature, the uniform gradient defines the most critical stress condition caused by warping." This is born out by the fact that numerical values of the stresses computed by Thomlinson's analysis are less than those of Westergaard.

The analyses of Westergaard and Thomlinson represent the major general theories that have been developed for the calculation of warping stresses in concrete pavements due to temperature gradients. Both theories are predicated on

full and intimate subgrade support. In [illegible] previous [illegible] that this assumption may be seriously in error, the reliability of the stresses computed on this basis is open to question. In 1943, on the basis of the test results from the Arlington investigation, Teller and Sutherland (53) adjusted Westergaard's formula for corner and edge loadings to account for reduced support caused by warping effects. Similar modification to the "corner formula" were made by Pickett in developing the Portland Cement Association formula (45), by Spangler (53), and by others (6, 32). These adjustments improved design concepts considerably; however, the authors believe that the general approach is conceptually inadequate. Westergaard's theory is still widely used to appraise the influence of subgrade modulus and slab thickness on the critical stresses. Current formulae suggest that the critical stresses will produce diagonal cracks at the corners, while the evidence previously cited clearly shows that the majority of cracks are now transverse. Finally, many investigators attempting to evaluate concrete pavements by means of measured strains and deflections have generated considerable confusion. As strain gauges have been applied without due cognizance of warping stresses already extant, and deflections have been measured without knowledge of the support conditions.

New Theory

In this paper equations are presented for the deflections and stresses in warped slabs of finite size, allowing for the condition that warping may result in a partially supported slab. Numerical solutions were obtained for the case of an unloaded slab (apart from its weight), which was used as a model for a highway pavement. Charts for deflections and stresses are given covering a wide variety of slab parameters and ambient temperature and moisture conditions. The influence of concrete quality, subgrade modulus, and slab thickness is evaluated; major discrepancies between these influences and current concepts derived

from Westergarrd's theory are highlighted. Finally, comparisons are made with available measured data and the validity of the new theory is assessed.

THEORY

Notation

w = deflection, positive in downward direction

q = uniformly distributed load

k = modulus of subgrade reaction

h = slab thickness

μ = Poisson's ratio

E = Young's modulus

D = flexural rigidity of the slab = $\frac{Eh^3}{12(1-\mu^2)}$

ℓ = radius of relative stiffness = $\sqrt[4]{D/k}$

α = linear coefficient of thermal expansion

T = equivalent temperature difference between upper and lower slab surfaces, length changes caused by moisture differences are transformed into equivalent temperature differences.

r = radial distance

a = slab radius

b = radial distance to point of zero deflection

$\beta = \frac{b}{\ell}$

$\rho = \frac{r}{\ell}$

$\nabla_r^2 = \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)$

$\nabla_r^4 = \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\nabla_r^2$

$V(r)^* = -D\left[\frac{d}{dr}\nabla_r^2 w\right]$ = shear at point r

$M(r)^* = -D\left[\frac{d^2w}{dr^2} + \frac{\mu}{r}\frac{dw}{dr} + \alpha(1+\mu)\frac{T}{h}\right]$ = radial bending moment at point r

---

* c.f. Melan, E., and Parkus, H., Warmespannungen," Springer-Verlag, Vienna (1953).

$\sigma(r)^{*} = -\frac{Eh}{2(1-\mu^2)}\left[\frac{M(r)}{D}\right]$ = normal radial stress at point r, positive denotes tension

σ_o = normal (radial) stress at center of slab

C_j = coefficients

$Z_1(\rho)$ = ber (ρ)

$Z_2(\rho)$ = -bei (ρ)

$Z_3(\rho) = -\frac{2}{\pi}$ kei (ρ)

$Z_4(\rho) = -\frac{2}{\pi}$ ker (ρ)

$Z_i'(\rho)$ = first derivative of $Z_i(\rho)$ with respect to ρ.

Formulation of Problem

The theoretical model chosen for analysis considers the bending of circular slabs of uniform thickness resting on homogeneous foundations whose reaction against the slab is proportional to the deflection. The only forces acting on the slab are due to its weight. The slab is subject to changes in temperature (and/or moisture) which increase linearly with depth but remain constant on any given planes parallel to the surface of the slab; consequently, the edges of the slab will tend to warp up. The slab is divided into two zones (Figure 1) with the inner portion (zone 1) being a uniformly loaded slab on an elastic foundation while the outer region (zone 2) is unsupported by the foundation. Conditions of continuity require that along the circle of radius, b, the deflection, slope, bending moment, and shear of both inner and outer portions be equal.

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* c.f. Melan, E., and Parkus, H., "Warmespannungen," Springer-Verlag, Vienna (1953).

As the loading and reactive forces are symmetrically distributed about an axis perpendicular to the slab through its center, both the slope and the shear at the center of the slab must be zero.

The assumptions, boundary conditions, and symmetry may be summarized as follows:*

a) $w_1'(0) = 0$

b) $V_1(0) = 0$

c) $w_1(b) = 0$

d) $V_2(a) = 0$

e) $M_2(a) = 0$

f) $w_2(b) = 0$

g) $w_1'(b) = w_2'(b)$

h) $M_1(b) = M_2(b)$

i) $V_1(b) = V_2(b)$

Solution of Problem

Assuming that deflections are small in comparison with the thickness of the slab, the following differential equations pertain to the supported and unsupported portions of the slab, respectively (49):

$$D\nabla_r^4 w = q - kw \qquad \text{(supported zone)},$$

and

$$D\nabla_r^4 w = q_r \qquad \text{(unsupported zone)}.$$

---

* Subscripts refer to zones (Figure 1)

Applying the conditions listed above to the governing differential equations, the following equations for stresses and deflections in the slab are obtained.*

For $0 < r \leq b$ or $0 < \rho \leq \beta$,

$$\sigma_\rho = \frac{Eh}{2(1-\mu^2)}\left[\frac{q}{k\ell^2}\left\{C_1\left[Z_2(\rho) - \frac{(1-\mu)Z_1'(\rho)}{\rho}\right] - C_2\left[Z_1(\rho) + \frac{(1-\mu)Z_2'(\rho)}{\rho}\right]\right\} + \alpha(1+\mu)\frac{T}{h}\right]$$

$$w_1(\rho) = \frac{q}{k}\left\{1 + C_1 Z_1(\rho) + C_2 Z_2(\rho)\right\}.$$

For $b \leq r \leq a$,

$$\sigma_r = \frac{Eh}{2(1-\mu^2)}\left\{C_6(\mu-1)\left(\frac{1}{r^2} - \frac{1}{a^2}\right) + \frac{qa^2}{8D}(1+\mu)\ln\frac{a}{r} - \frac{q(3+\mu)}{16D}(a^2 - r^2)\right\}$$

$$w_2(r) = C_5 + C_6 \ln r + C_7 r^2 + \frac{qr^4}{64D} - \frac{qa^2r^2}{8D}\ln r.$$

At $r = \rho = 0$,

$$\sigma_0 = \frac{Eh}{2(1-\mu^2)}\left[-\frac{C_2 q}{2k\ell^2}(1+\mu) + \alpha(1+\mu)\frac{T}{h}\right].$$

where

$$C_1 = -\frac{1 + C_2 Z_2(\beta)}{Z_1(\beta)}$$

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* These equations are derived in a paper by the authors entitled "Analysis of Concrete Slabs on Ground," submitted for publication in the Journal, Soil Mechanics and Foundations Division, ASCE. Solutions are also presented there for superimposed peripheral loads, and for concentrated loads at the center of the slab.

$$\frac{C_6(\mu-1)}{a^2} + 2C_7(1+\mu) - \frac{qa^2}{16D}\left[4(1+\mu)\ln a + (3+\mu)\right] + \alpha(1+\mu)\frac{T}{h} = 0$$

$$C_5 + C_6 \ln b + C_7 b^2 - \frac{qa^2b^2\ln b}{8D} + \frac{qb^4}{64D} = 0$$

$$\frac{C_6}{b} + 2C_7 b - \frac{qa^2b}{8D}(2\ln b + 1) + \frac{qb^3}{16D} - \frac{q}{K\ell}\left[C_1 Z_1'(\beta) + C_2 Z_2'(\beta)\right] = 0$$

$$\frac{C_6}{b^2}(\mu-1) + 2C_7(1+\mu) - \frac{qa^2}{8D}\left[2(1+\mu)\ln b + (3+\mu)\right] + \frac{qb^2}{16D}(3+\mu)$$

$$- \frac{q}{K\ell^2}\left\{C_1\left[Z_2(\beta) - \frac{(1-\mu)Z_1'(\beta)}{\beta}\right] - C_2\left[Z_1(\beta) + \frac{(1-\mu)Z_2'(\beta)}{\beta}\right]\right\} = 0$$

$$C_2 = -\left\{\frac{Z_2'(\beta) - Z_1(\beta)\left[a^2-b^2\right]\frac{1}{2b\ell}}{Z_2(\beta)Z_2'(\beta) + Z_1'(\beta)Z_1(\beta)}\right\}.$$

An examination of these equations shows that a prohibitive amount of work would be required to eliminate the constants of integration to obtain an explicit expression for either the stresses or deflections in terms of the slab parameters. Accordingly, the problem was programmed for the Datatron 204 digital computer at Purdue University. As a part of the solution, subroutines were developed for the ber and bei functions and their derivatives. Figure 2 is the flow chart for the main program.

[illegible]

[illegible] radial stresses for [illegible]

[illegible]

[illegible]

[

DISCUSSION OF RESULTS

Examination of Assumptions

In the derivation of the theory it was assumed that the slab had a free edge boundary. This was believed to be justifiable as the only loads considered are those due to the weight of the slab; separate slabs would be subjected to the same effective gradient, thus resulting in little shear or moment transfer. Field deflection measurements reported by Hveem (27) appear to substantiate this assumption.

Highway and airport pavement slabs are rectangular in shape rather than circular. The circular shape was selected for study because solutions could be obtained in closed form for finite slab sizes. It is pertinent, therefore, to examine the effects of slab shape on the developed critical stresses.

Figure 8 represents a plot of the ratios of the normal stress at the center of a circular slab to the normal stress at the center of an infinitely long slab, as given by Westergaard, where the diameter of the circular slab is equal to the width of the long slab. As noted earlier, Westergaard's solution was predicated on the assumption that the subgrade provides full and continuous support to the pavement at all times; hence, the stresses in the circular slabs were computed for full subgrade support. For slab diameters larger than those corresponding to W/ℓ ratios of 8, the shape of the slab does not appear to be of much importance. The comparisons between measured deflections of warped rectangular slabs to those computed for circular slabs (reported below), lend further weight to the validity of this observation.

Typical slab diameters corresponding to W/ℓ ratios of 6 are tabulated below:

			K = 200 pci		K = 700 pci	
E	μ	h	W=2a	T	W=2a	T
3×10^6 psi	0.15	8"	19.0	4.36°F	13.9	2.33°F
3×10^6 psi	0.15	10"	22.4	4.87°F	16.4	2.61°F
3×10^6 psi	0.15	12"	25.7	5.34°F	18.8	2.85°F

From the above tabulation, it is seen that the maximum equivalent temperature difference for which the Westergaard theory is applicable is very much lower than those commonly encountered in practice.

Comparisons with Available Measurements

Reference was made previously to measured slab deflections obtained by Hatt. Curve 2 (Figure 9) represents the maximum position of the slab during the drying cycle, whereas curve 4 depicts the maximum position of the slab resulting from drying the slab completely and then introducing water into the subbase and maintaining its level at the lower surface of the slab. The deflection measurements were taken at times when the temperatures were constant throughout the slab. No information was given on the modulus of subgrade reaction of the foundation or on the modulus of elasticity of the concrete. To compare the measured deflections with those computed on the basis of the proposed theory, parameters thought to be appropriate to the conditions described by Hatt were chosen. Attention is directed to the computed effective temperature differences (T), shown in Figure 9, which in this case reflect the effects of moisture gradients only.

The solid curve in Figure 10 represents the longitudinal deflection curve of a reactangular slab (12 x 24 feet), six inches thick, as measured at the Portland Cement Association Laboratories, Skokie, Illinois. This slab was subjected to both temperature and moisture gradients. Comparisons between theoretical and observed deflections for this slab are of special interest, as

both the modulus of subgrade reaction (K = 80 pci) of the base and the dynamic modulus of elasticity (4,500,000 psi) of the concrete were determined. Reducing the dynamic modulus of elasticity to an equivalent static modulus (63), and using an equivalent circular slab 24 feet in diameter, and dotted curve in Figure 10 was obtained.

The correspondence between the computed and measured deflections, as shown in Figures 9 and 10, offers substantial additional evidence regarding the validity of the proposed theory and of the minor importance of the shape of the slab on the critical stresses and deflections.

Effective Gradients

The results shown in Figures 5A to 5L show that effective temperature differences between slab surfaces of approximately 40 degress F are needed to produce tensile stresses which approach the modulus of rupture of the slabs. Observations (34, 58) indicate that the maximum temperature gradient in highway pavement slabs (bottom warmer than top) is on the order of 1-1/2 to 2 degrees F. per inch of slab thickness (for 9 inch slabs). Accordingly, critical warping conditions can develop only if appreciable moisture gradients are present.

As demonstrated by Hatt (23), moisture differences between the top and bottom of slabs causes warping in much the same manner as temperature variations. However, very little known of the developed moisture gradients, due to the lack of suitable instrumentation capable of measuring transient moisture gradients in concrete slabs.

Using fragments broken from specimen slabs and determining the moisture content by drying in the laboratory, Teller and Sutherland (58), reported a seasonal variation of moisture content of 0.3 percent. However, they state in their conclusions:

> There is a cyclic variation in slab length that is entirely dissociated from temperature changes. The annual variation in the length of the test sections from causes other than temperature changes is approximately equivalent to that caused by a temperature change of 30°F., and the maximum length occurs during the late winter when the ground moisture content is greatest.

Hveem (27) showed that changes in moisture content for 30 oven-dried thin concrete discs (obtained from pavement cores), which were allowed to soak for 7 days, evidenced a greater percent expansion for 2/3 of the specimens than did a 90°F. increase in temperature. Hveem concluded that, "In the majority of cases, however, it appears that under road conditions the expansions due to moisture and to temperature would be approximately the same." Shideler (50) and Kalousek (31) have demonstrated that a 2 percent change in moisture in the critical range can account for over 70 percent of the total volume change of concrete masonry units (cf. Figure 11). Thus, large volume changes may be anticipated in highway pavements with but small moisture differences between slab surfaces. Although many factors, such as aggregates, water-cement ratios, curing, carbonation, etc., affect the moisture content-volume change relationship, the available data strongly indicate that temperature and moisture gradients encountered in practice may occasionally combine to produce tensile stresses which exceed the modulus of rupture of the concrete.

It is recognized that permanent distortions of concrete slabs may result from residual differential shrinkage and carbonation (50). However, these effects cannot be evaluated at the present time.

Effects of Slab Dimensions, Quality of Concrete Subgrade Modulus, and Equivalent Temperature Gradients on the Critical Stresses Developed

To compare the influence of slab thickness on the resulting stresses, information is required on the relationship between ambient conditions, slab thickness, and developed critical equivalent temperature gradients. As such data are not yet available, comparisons were made in two ways:

1. The maximum equivalent temperature difference b tween top and bottom of the slab is kept constant. The influence of slab size, thickness, temperature gradient, quality of concrete, and modulus of subgrade reaction on the ratio of the developed stress to the modulus of rupture is shown in Figure 12.

2. The maximum equivalent temperature gradient in the slab is kept constant. The influence of slab size, thickness, temperature gradient, quality of concrete, and modulus of subgrade reaction on the ratio of the developed stress to the modulus of rupture is shown in Figure 13.

From Figures 12 and 13 it may be seen that the benefits to be gained from increasing the slab thickness are dependent on the equivalent temperature gradients that can develop under ambient conditions. In fact, these comparisons clearly demonstrate that the effects of any given parameter cannot be isolated from the other factors. For example, for a 40-ft. diameter slab, an 8-inch thick slab shows a decrease in stress (σ_c/M_r) with increasing modulus of subgrade reaction (k); while for a 20-ft. diameter slab, there is a substantial increase in stress with increasing k. A 12-inch slab is closer to failure under an equivalent temperature difference of 40°F. using higher quality concrete ($E = 5 \times 10^6$ psi) if the k values are high, while the reverse is the case if the k values are low. Thus, it would be hazardous to propose any generalized design recommendations until

the effects of ambient conditions on the critical equivalent temperature gradients have been determined. It will also be necessary to know the effects of creep on the stresses developed by thermal and moisture gradients, shrinkage and carbonation. For these reasons, the new formulae are presented at this time not for use as design criteria, but primarily as a contribution to a better understanding of slab action and to point the way towards more significant field measurements that will ultimately lead to improved rational design procedures.

Effects of Warping

The foregoing results show that warping can induce stresses of sufficient magnitude to cause cracking of concrete highway pavements. As the stresses are more critical in the longitudinal direction, transverse cracks in existing pavements can result from this warping, even in the absence of superimposed loads due to traffic. Due to complex interactions between the many factors involved, the discussion of effects of traffic loads must be delayed until numerical solutions of the derived equations for super-imposed loads are obtained.

Warping of pavements is also of importance from the standpoint of the interpretation of deflection and strain measurements induced by external loads. The relationships between stress, strain, and deflection registered at a point in the pavement are greatly influenced by the position of the pavement relative to the subbase at the time of loading, and by the stresses already extant in the slab as a result of restraint to length changes induced by thermal and moisture differentials. Accordingly, unless these effects are properly accounted for, interpretations of pavement performance on the basis of relationships between applied loads and measured strains or deflections cannot be made on a rational basis.

CONCLUSIONS

1. A theory has been developed whereby the stresses, deflections, and degree of support of slabs, subject to warping caused by ambient temperature and/or moisture gradients, can be computed for finite slab sizes.

2. Comparisons with available theories for fully supported slabs, and between computed and measured deflections for partly supported slabs, indicate that, for conventional slab sizes, the shape of the slab has only a minor influence on the critical stresses.

3. The maximum equivalent temperature differences between slab surfaces at which a warped slab can maintain full contact with its support are much lower than those frequently encountered in practice.

4. Comparisons with available measurements show, that for the case of an unloaded slab (apart from its weight) with a free edge boundary, the theory is applicable to concrete pavements even if warping is sufficiently severe to result in a partially supported slab.

5. Considerations of the effects of warping are a necessary prerequisite to the interpretation of pavement performance on the basis of measured strains and/or deflections of the slab.

6. In the critical shrinkage range, a moisture gradient of one percent can cause warping equivalent to a 20 degree F. temperature gradient. Accordingly, cracking may occur at small temperature gradients if moisture differences between slab surfaces are only a few percent.

7. For partially supported slabs, complex interactions between the many factors involved preclude generalizations regarding the effects of slab thickness, subgrade support, and concrete quality on the developed stresses until a number of gaps in our knowledge have been closed.

8. The theory shows considerable promise as a useful tool for the analysis of a number of problems concerning the behavior of concrete slabs on ground. Its general validity can be appraised by means of experiments in which temperature and moisture gradients are carefully controlled, and reliable, simultaneous measurements of the deflections, strains, and degree of support are obtained with reference to the initial unwarped condition.

Data are required on the critical temperature and moisture gradients that develop under ambient conditions, for slabs of different thicknesses, before improved design procedures can be recommended. The effects of creep, residual shrinkage, and carbonation must also be evaluated.

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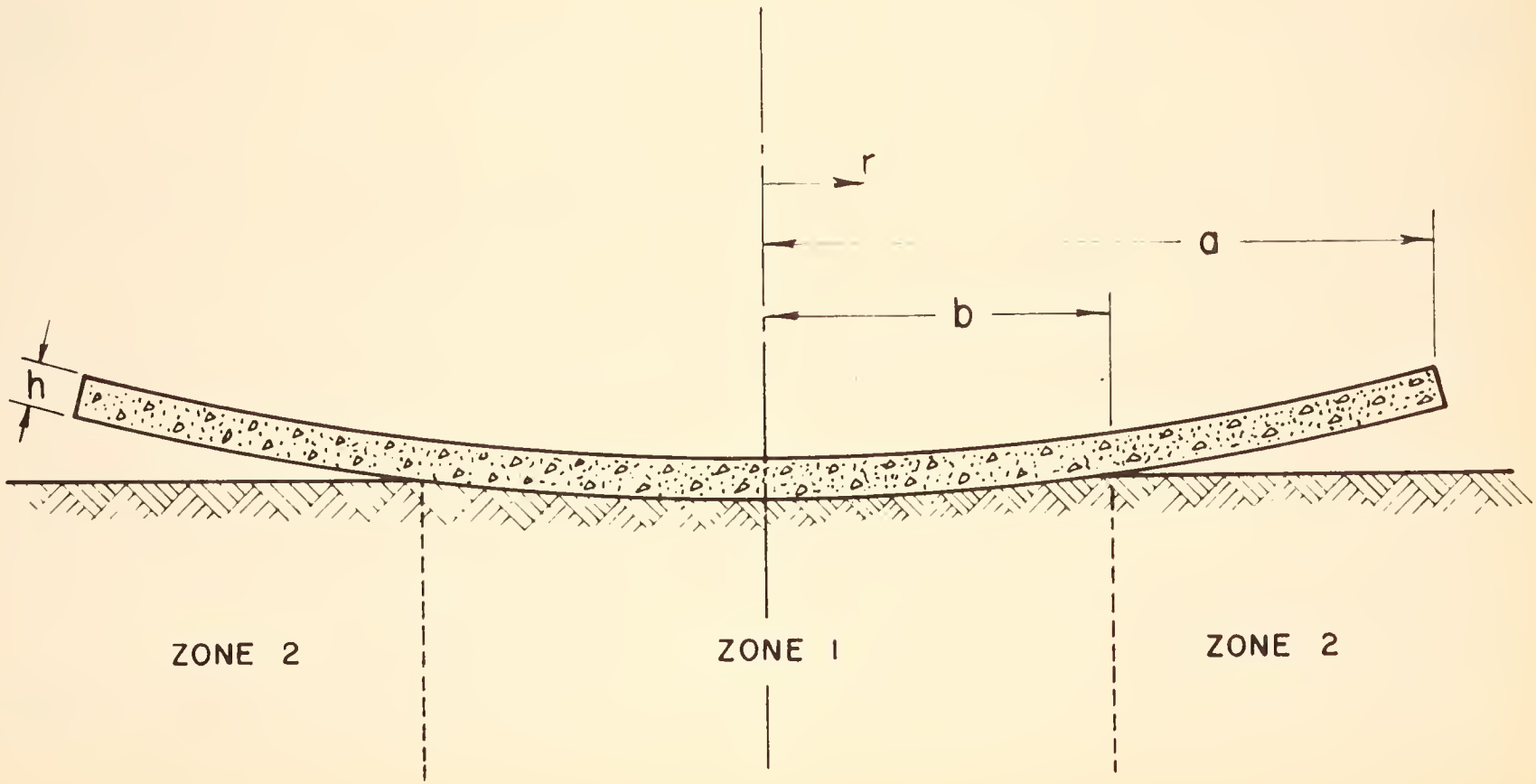
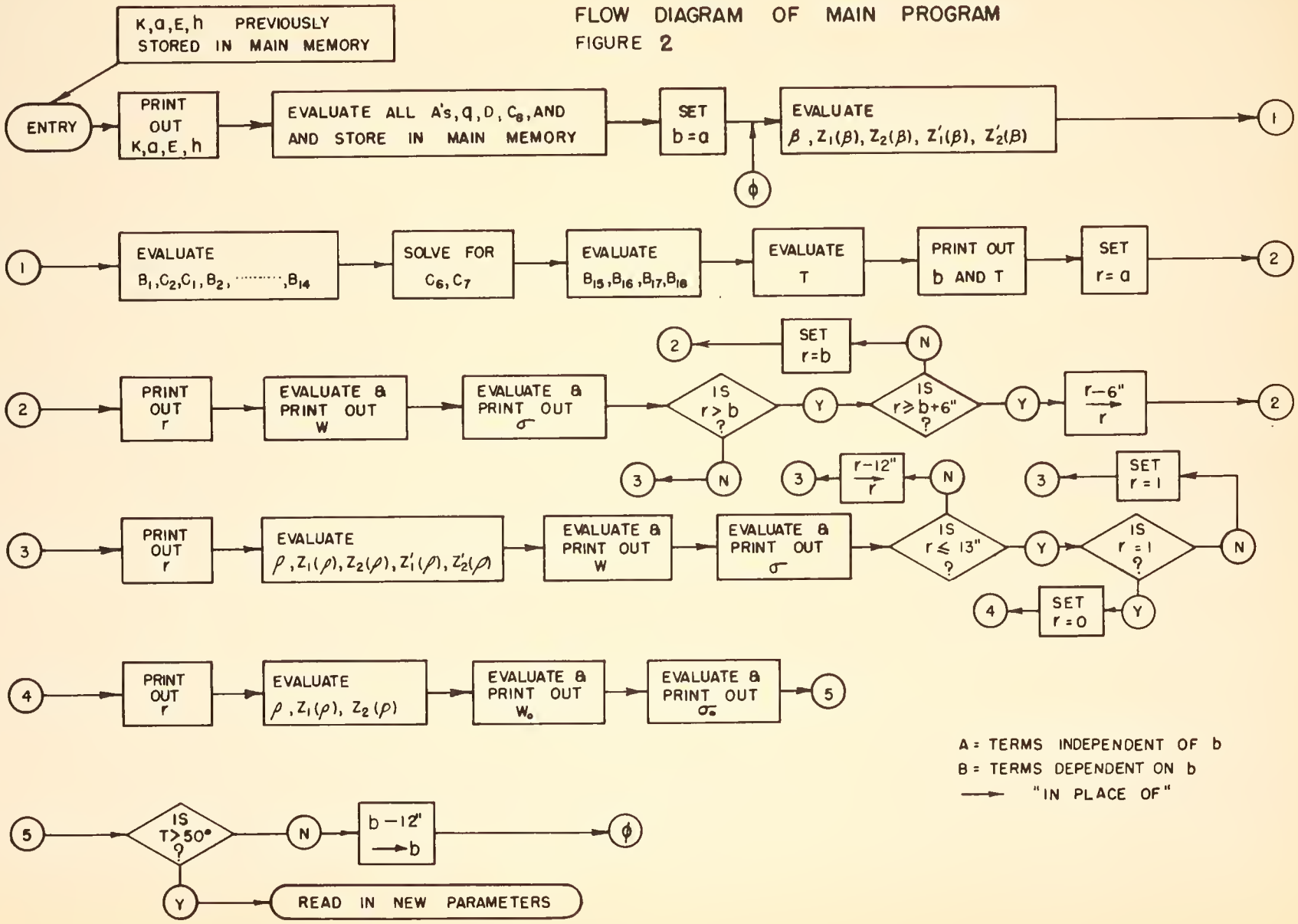
SIMPLIFIED DIAMETRAL SECTION OF WARPED SLAB

FIGURE 1



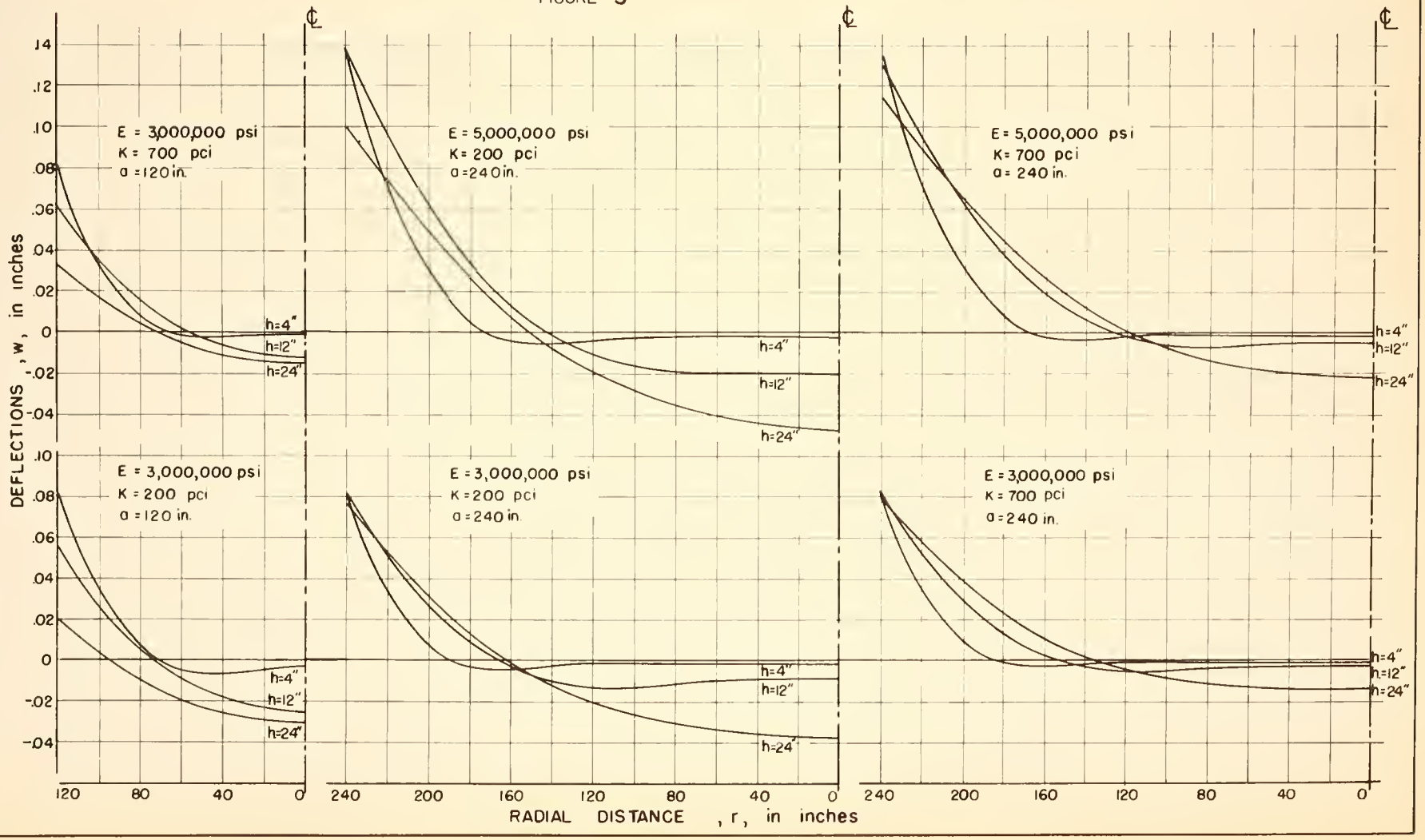
FLOW DIAGRAM OF MAIN PROGRAM

FIGURE 2



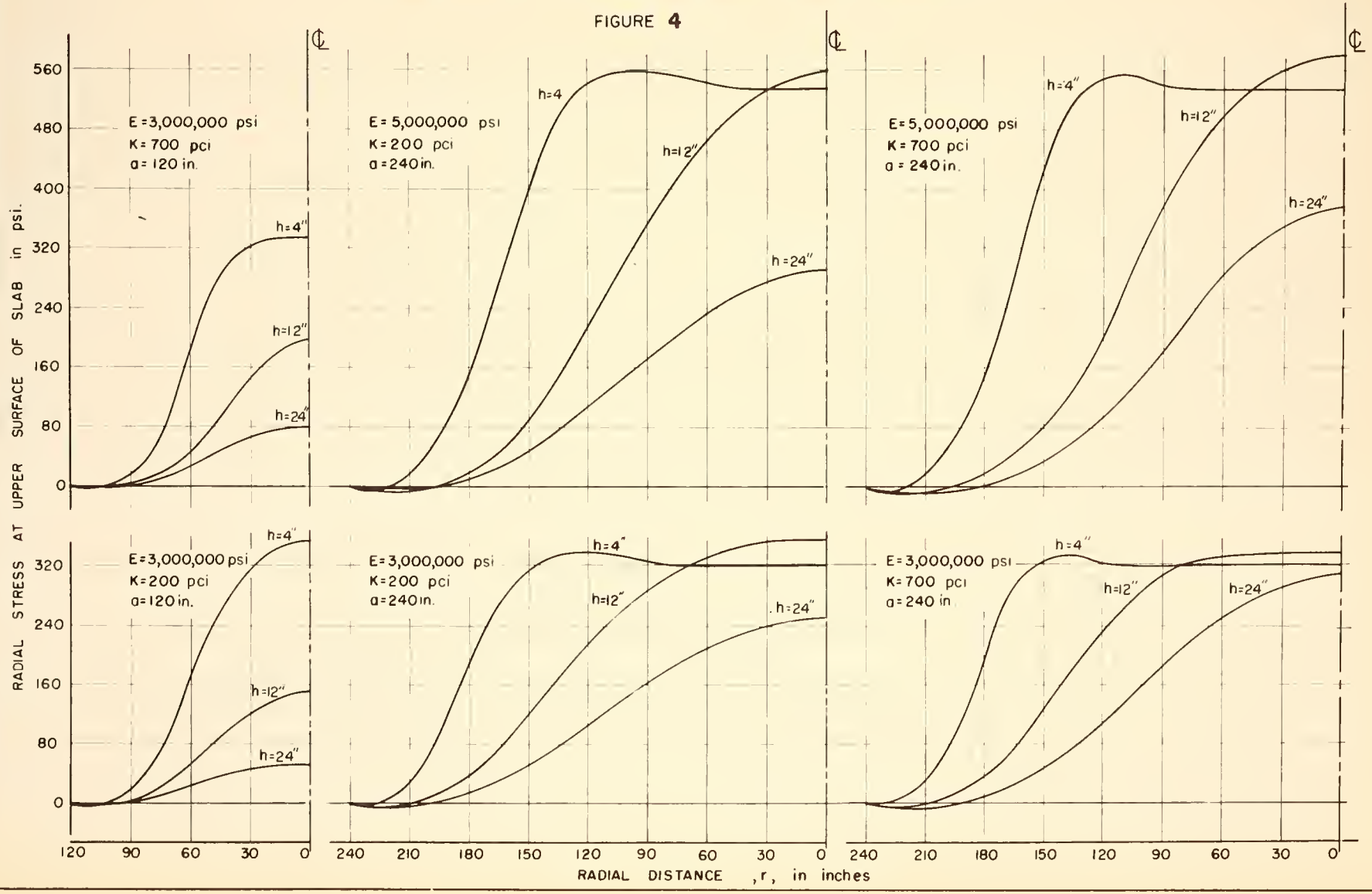
REPRESENTATIVE DEFLECTION CURVES FOR AN EFFECTIVE TEMPERATURE DIFFERENCE OF 30° F. BETWEEN SLAB SURFACES

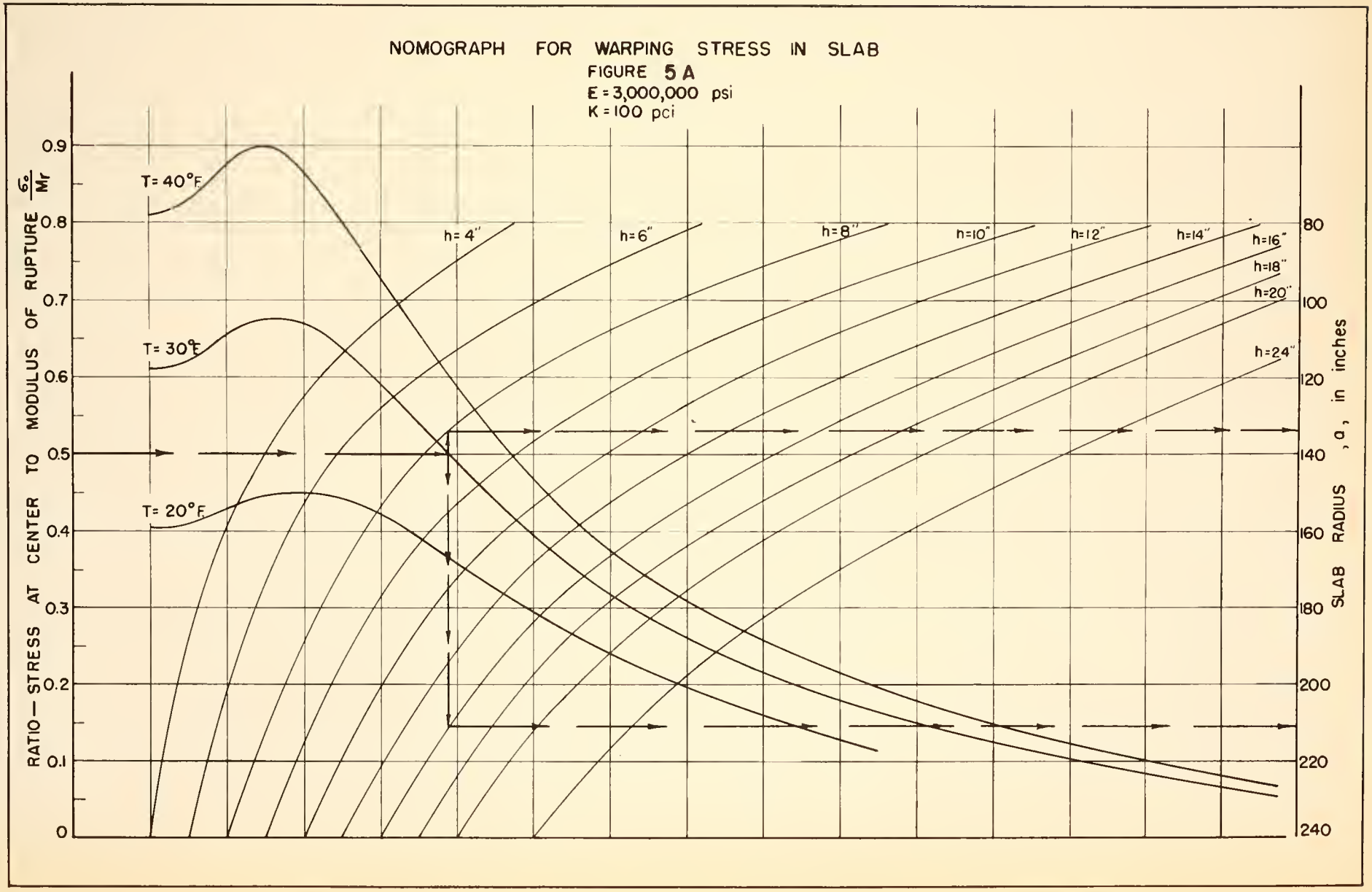
FIGURE 3

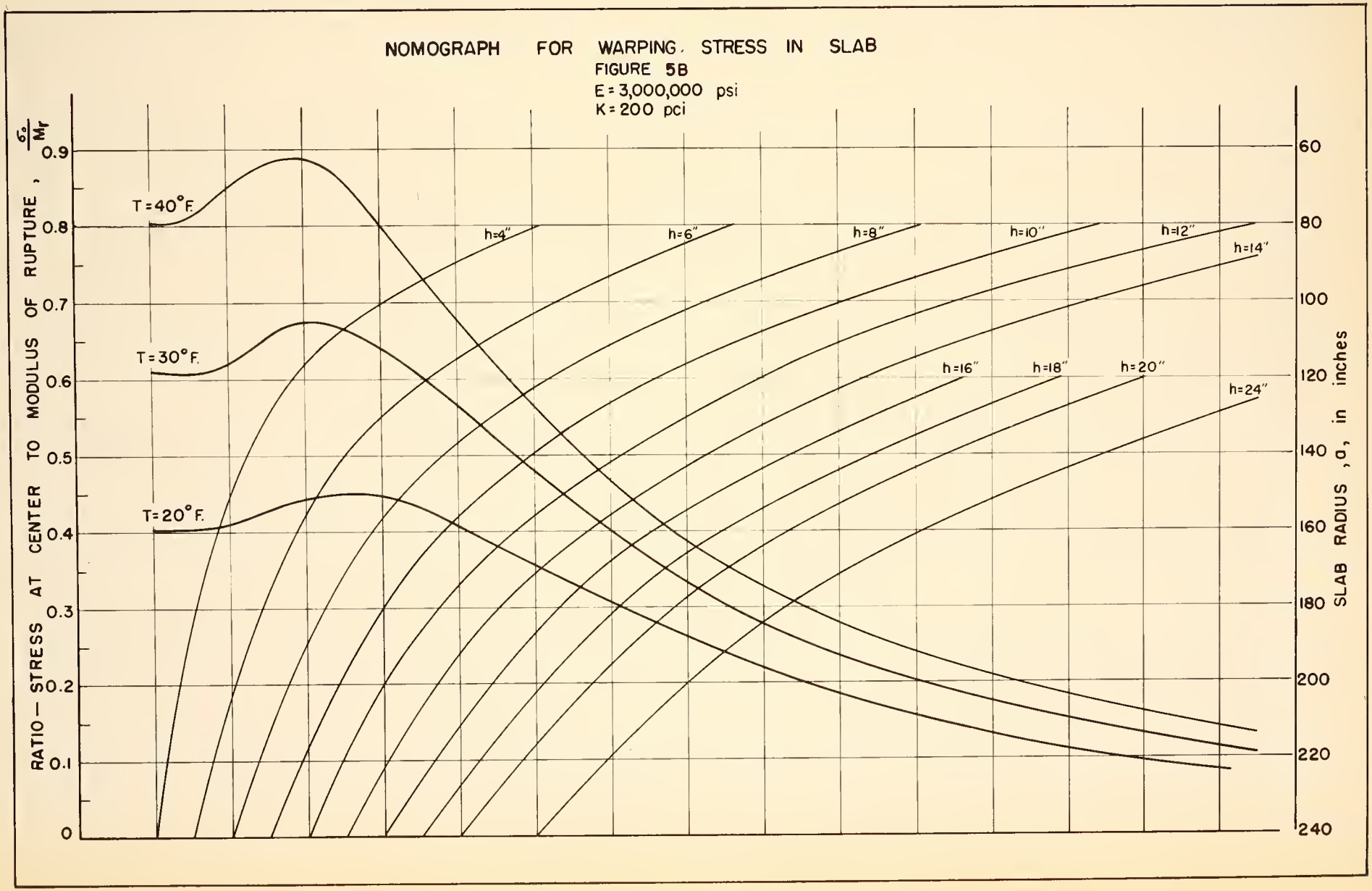


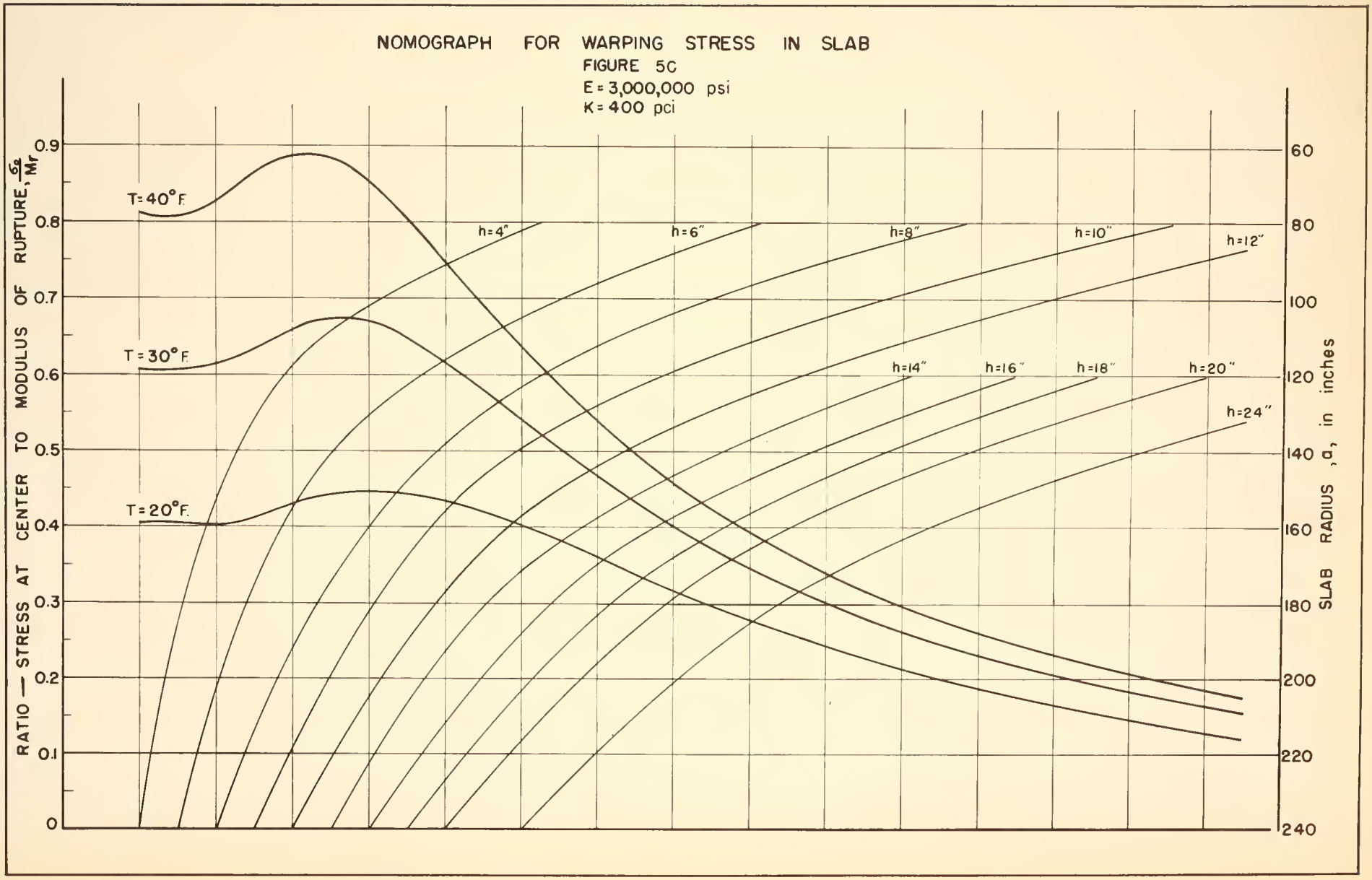
REPRESENTATIVE RADIAL STRESSES FOR AN EFFECTIVE TEMPERATURE DIFFERENCE OF 30° F. BETWEEN SLAB SURFACES

FIGURE 4



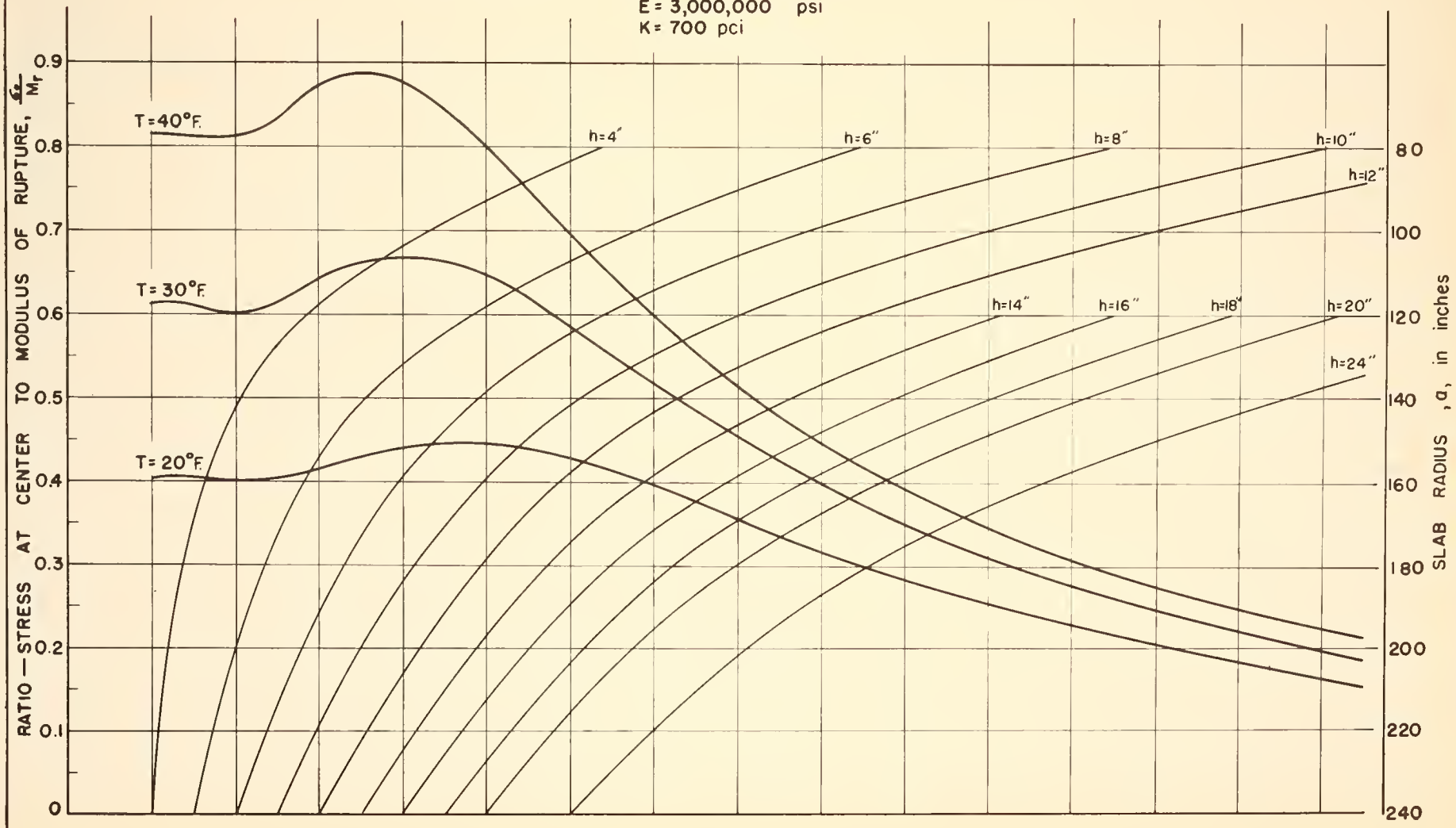

NOMOGRAPH FOR WARPING STRESS IN SLAB
FIGURE 5 A
E = 3,000,000 psi
K = 100 pci
RATIO — STRESS AT CENTER TO MODULUS OF RUPTURE $\frac{\sigma_o}{M_r}$
SLAB RADIUS, a, in inches
T= 40°F
T= 30°F
T= 20°F
h= 4"
h= 6"
h= 8"
h=10"
h=12"
h=14"
h=16"
h=18"
h=20"
h=24"
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
80
100
120
140
160
180
200
220
240


NOMOGRAPH FOR WARPING STRESS IN SLAB
FIGURE 5B
E = 3,000,000 psi
K = 200 pci
RATIO — STRESS AT CENTER TO MODULUS OF RUPTURE, $\frac{\sigma_o}{M_r}$
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
T = 40° F.
T = 30° F.
T = 20° F.
h=4"
h=6"
h=8"
h=10"
h=12"
h=14"
h=16"
h=18"
h=20"
h=24"
60
80
100
120
140
160
180
200
220
240
SLAB RADIUS, a, in inches


NOMOGRAPH FOR WARPING STRESS IN SLAB
FIGURE 5C
E = 3,000,000 psi
K = 400 pci
RATIO — STRESS AT CENTER TO MODULUS OF RUPTURE, σ/Mr
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
T=40°F.
T=30°F.
T=20°F.
h=4"
h=6"
h=8"
h=10"
h=12"
h=14"
h=16"
h=18"
h=20"
h=24"
60
80
100
120
140
160
180
200
220
240
SLAB RADIUS, a, in inches

NOMOGRAPH FOR WARPING STRESS IN SLAB

FIGURE 5D
E = 3,000,000 psi
K = 700 pci

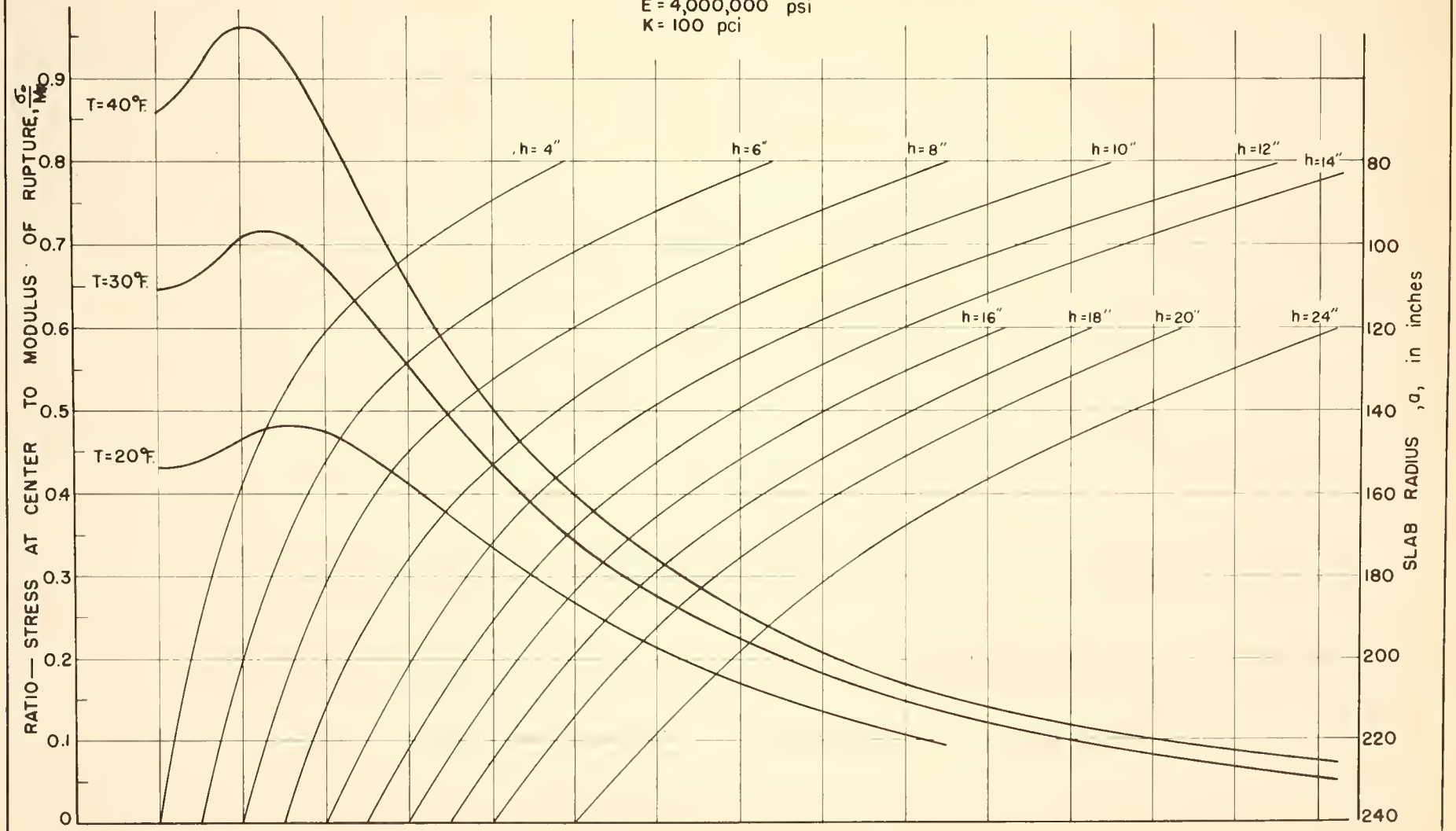


NOMOGRAPH FOR WARPING STRESS IN SLAB

FIGURE 5E

E = 4,000,000 psi

K = 100 pci

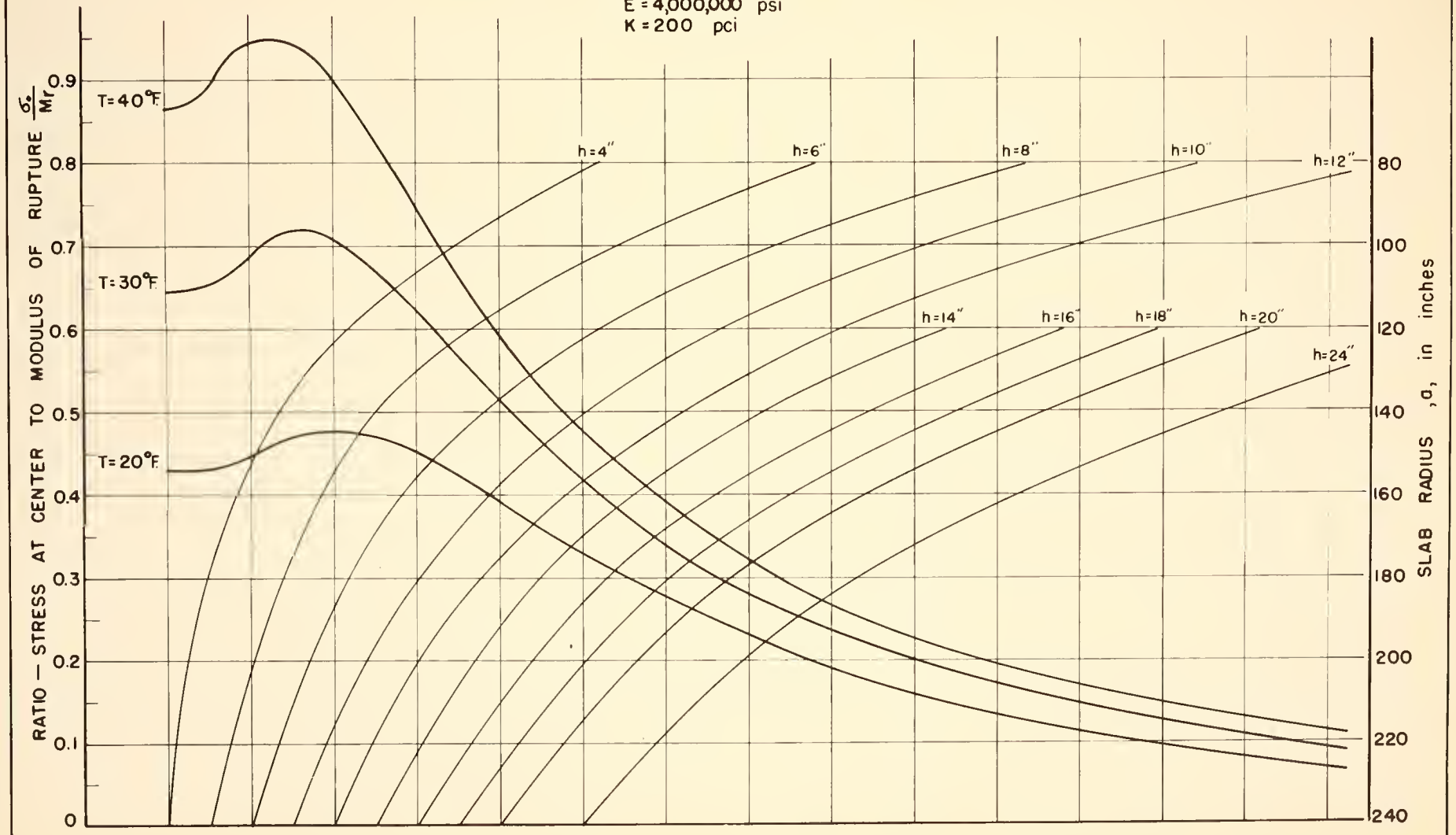


NOMOGRAPH FOR WARPING STRESS IN SLAB

FIGURE 5F.

E = 4,000,000 psi

K = 200 pci

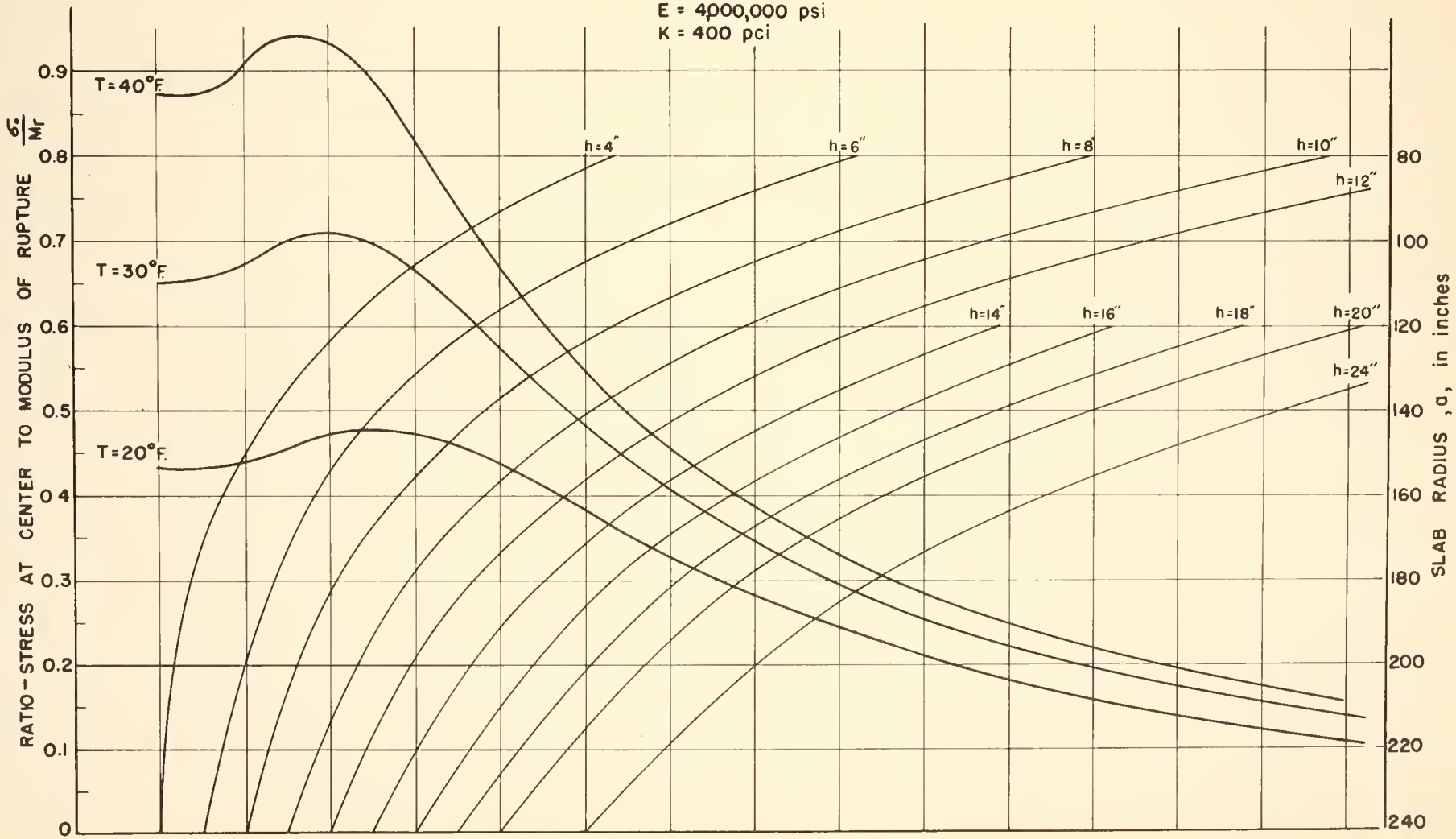


NOMOGRAPH FOR WARPING STRESS IN SLAB

FIGURE 5G

E = 4,000,000 psi

K = 400 pci

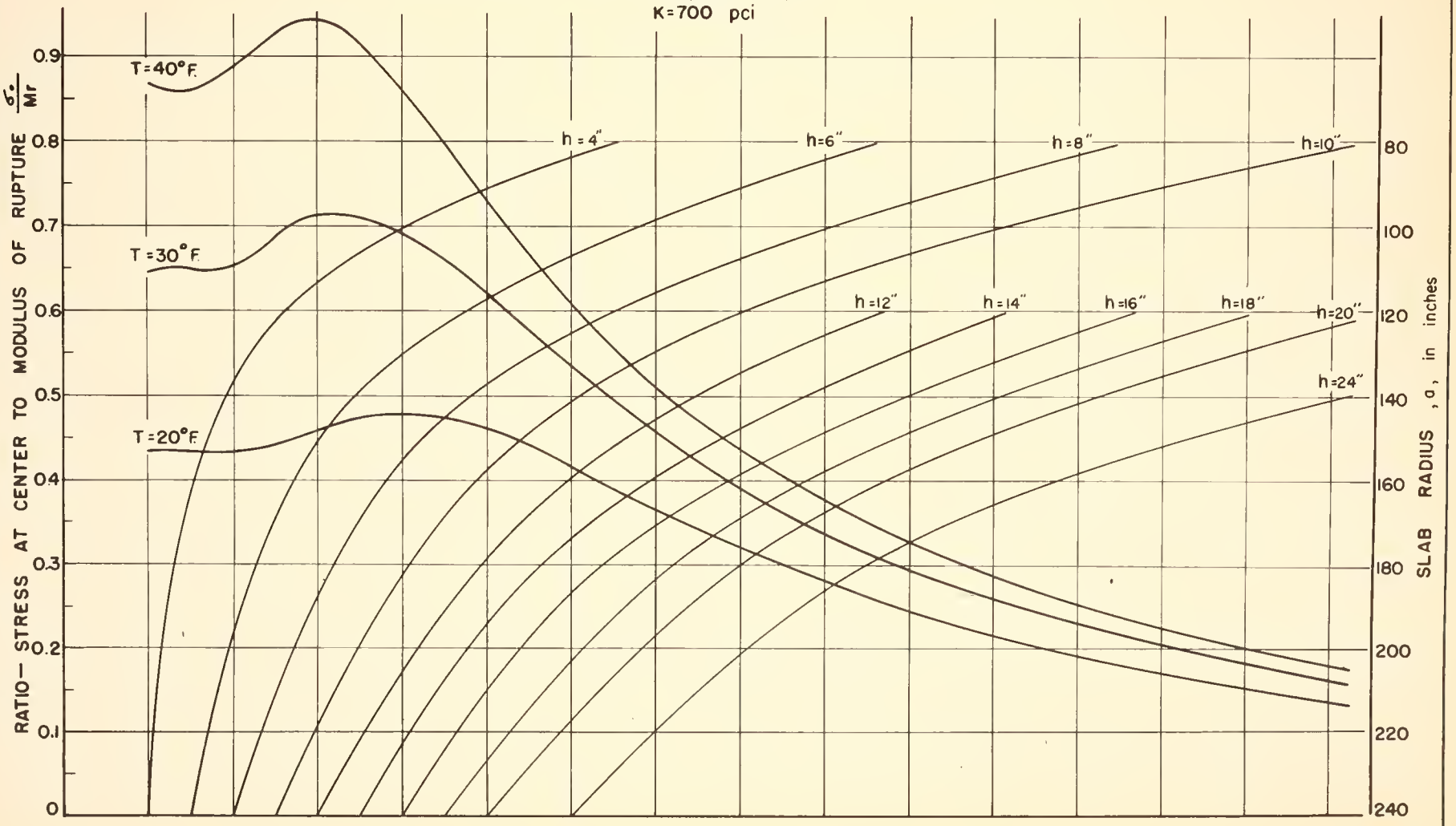


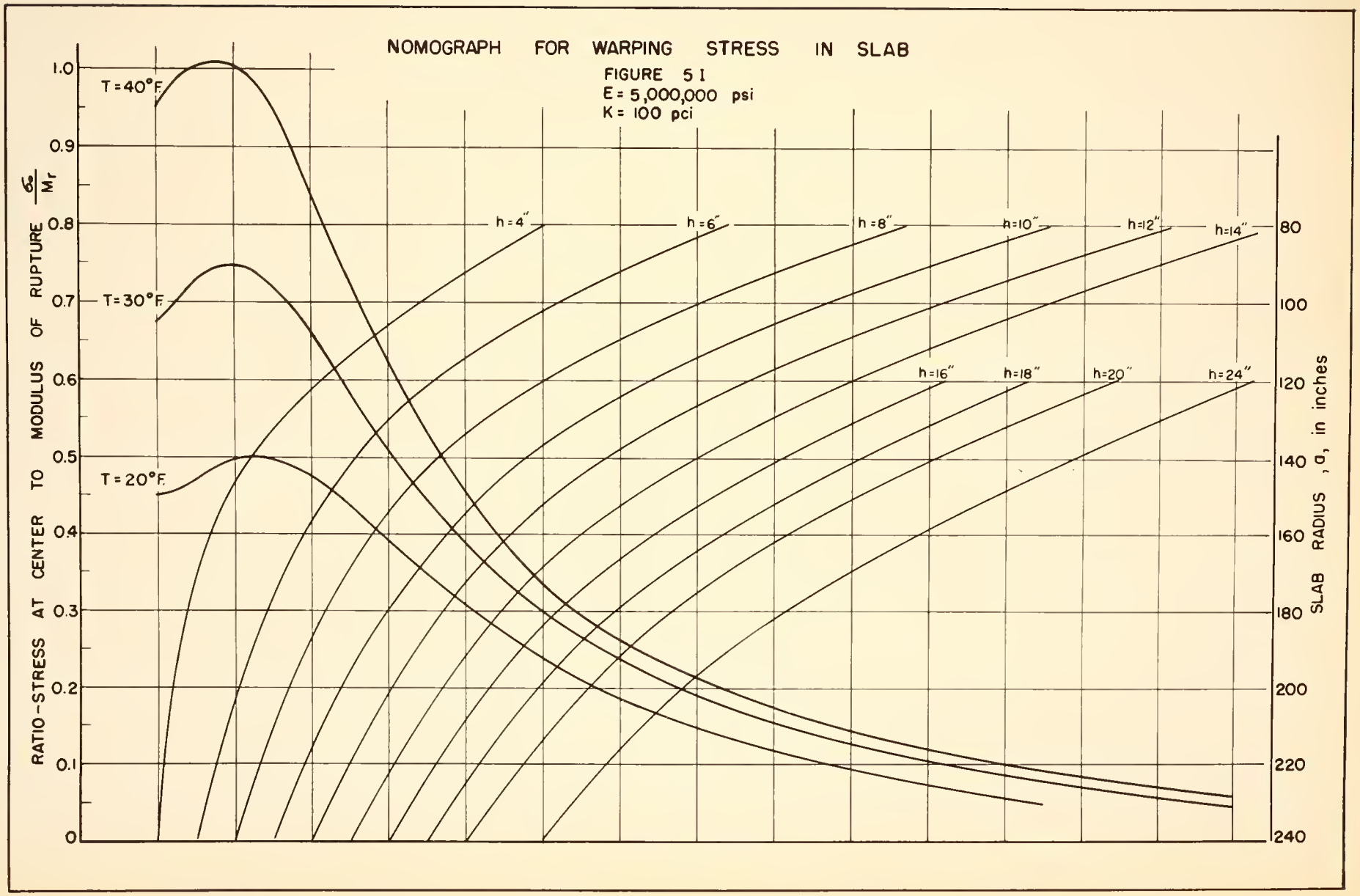
NOMOGRAPH FOR WARPING STRESS IN SLAB

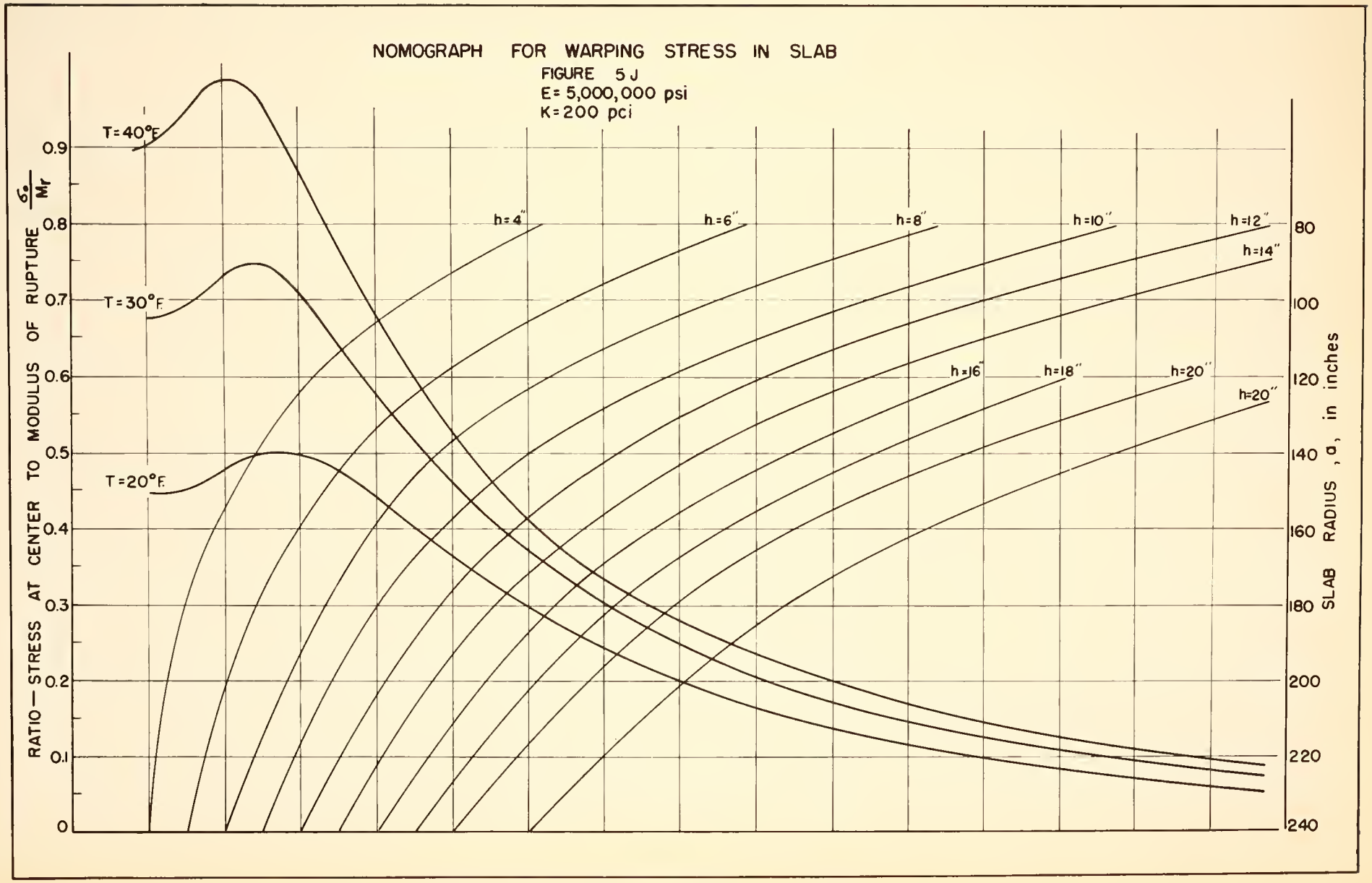
FIGURE 5H

E=4,000,000 psi

K=700 pci




NOMOGRAPH FOR WARPING STRESS IN SLAB
FIGURE 51
E = 5,000,000 psi
K = 100 pci
RATIO-STRESS AT CENTER TO MODULUS OF RUPTURE $\frac{\sigma_c}{M_r}$
SLAB RADIUS, a, in inches
T = 40°F.
T = 30°F.
T = 20°F.
h = 4"
h = 6"
h = 8"
h = 10"
h = 12"
h = 14"
h = 16"
h = 18"
h = 20"
h = 24"
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
80
100
120
140
160
180
200
220
240


NOMOGRAPH FOR WARPING STRESS IN SLAB
FIGURE 5 J
E= 5,000,000 psi
K=200 pci
RATIO — STRESS AT CENTER TO MODULUS OF RUPTURE $\frac{\sigma_o}{M_r}$
SLAB RADIUS, a, in inches
T=40°F
T=30°F
T=20°F
h=4"
h=6"
h=8"
h=10"
h=12"
h=14"
h=16"
h=18"
h=20"
h=20"
0
0.1
0.2
0.3
0.4
0.5
0.6
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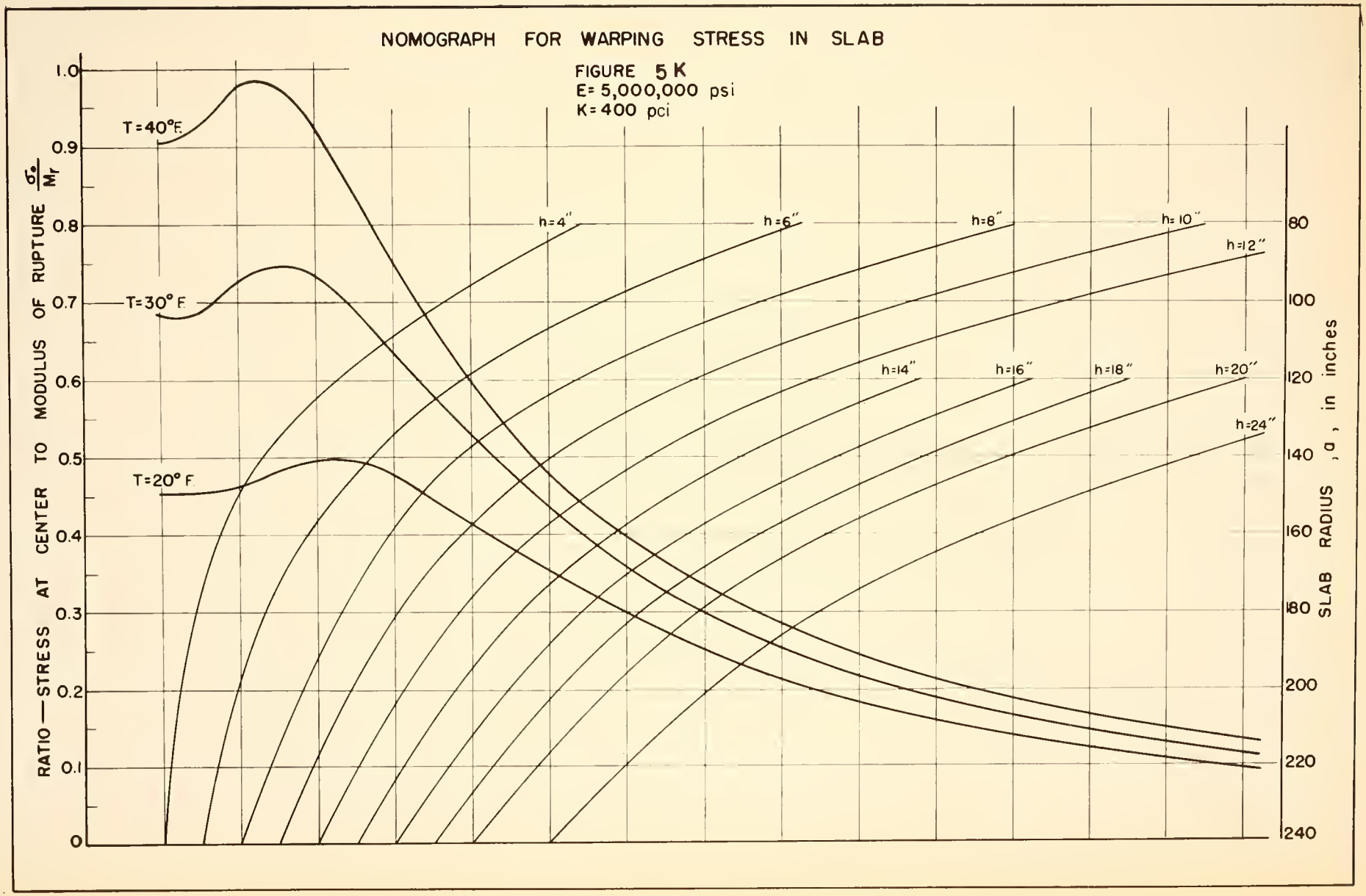
0.8
0.9
80
100
120
140
160
180
200
220
240

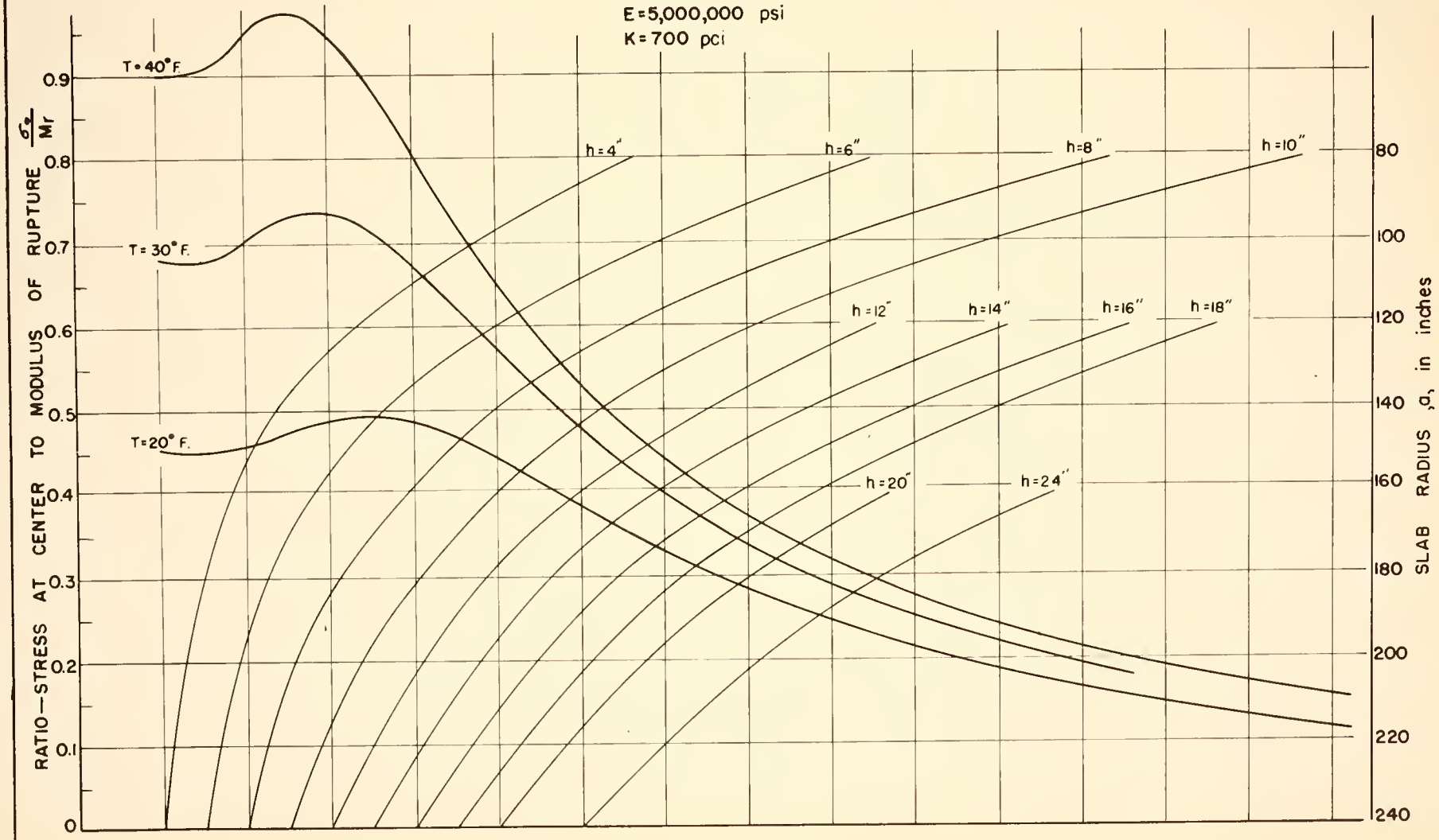

NOMOGRAPH FOR WARPING STRESS IN SLAB
FIGURE 5 K
E= 5,000,000 psi
K= 400 pci
RATIO — STRESS AT CENTER TO MODULUS OF RUPTURE $\frac{\sigma_o}{M_r}$
SLAB RADIUS, a, in inches
T=40°F
T=30° F
T=20° F
h=4"
h=6"
h=8"
h=10"
h=12"
h=14"
h=16"
h=18"
h=20"
h=24"
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
80
100
120
140
160
180
200
220
240

NOMOGRAPH FOR WARPING STRESS IN SLAB

FIGURE 5L

E = 5,000,000 psi

K = 700 pci

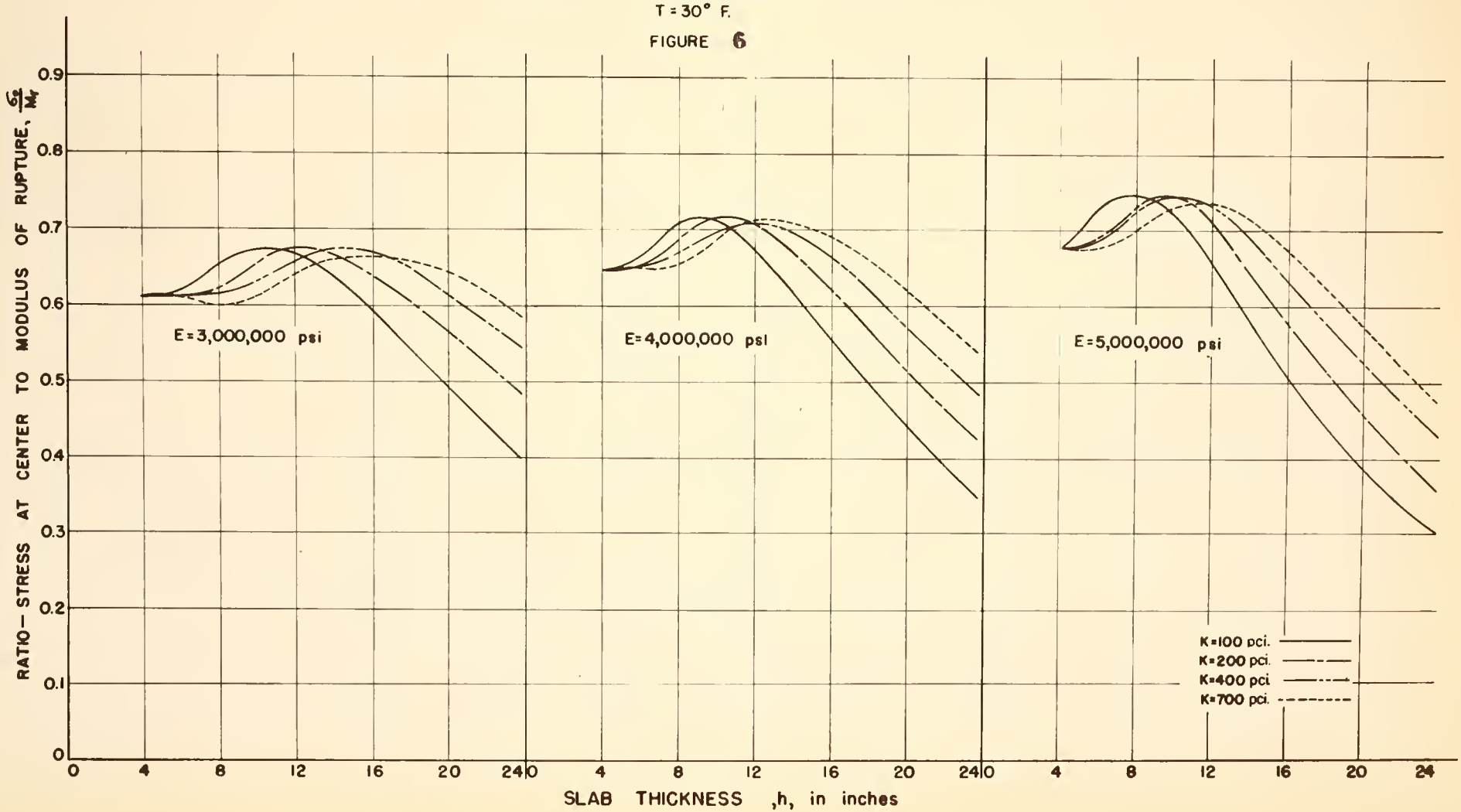


INFLUENCE OF MODULI OF ELASTICITY

a = 240 inches

T = 30° F.

FIGURE 6

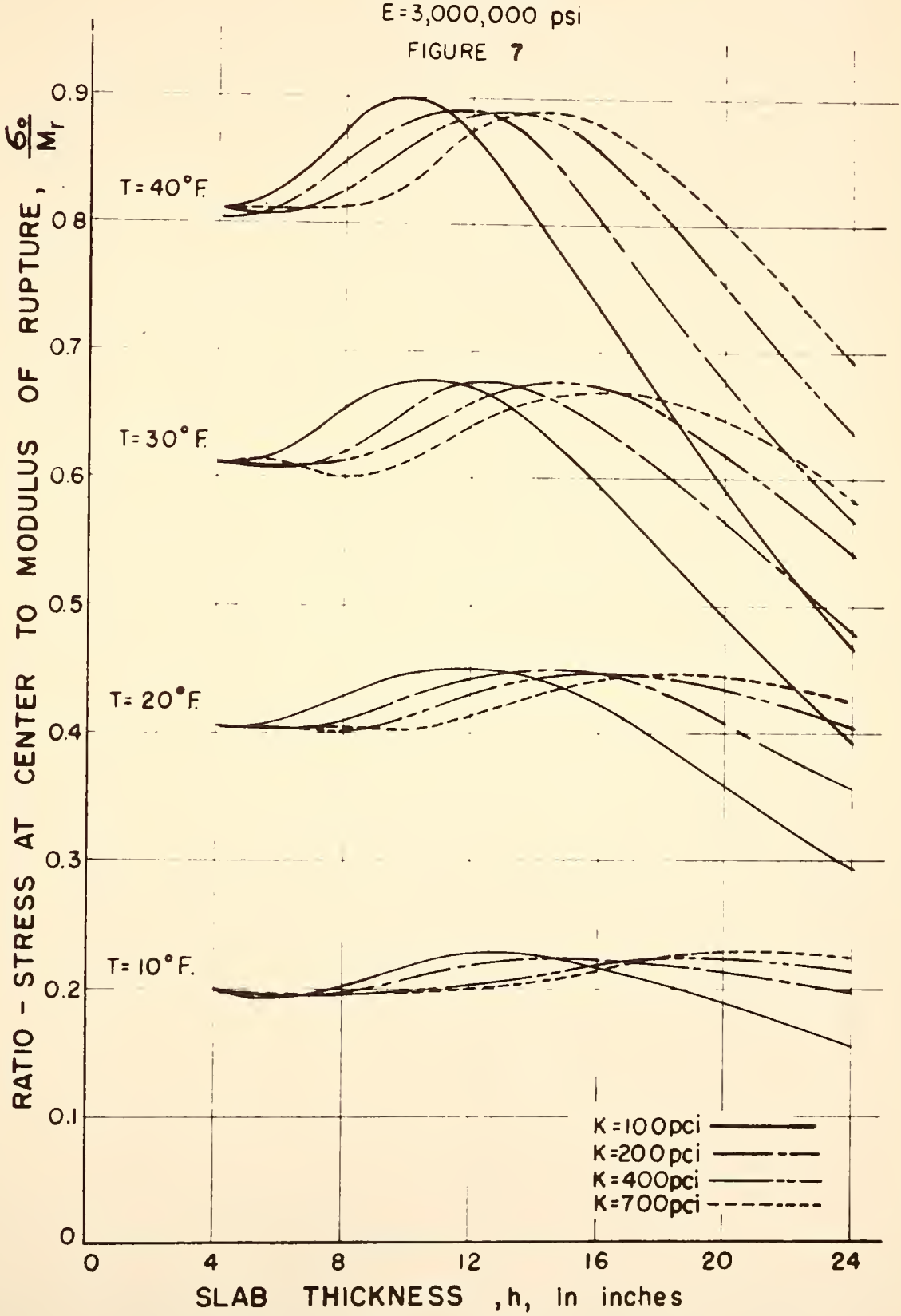


INFLUENCE OF VARYING EFFECTIVE TEMPERATURE DIFFERENCES

a = 240 inches

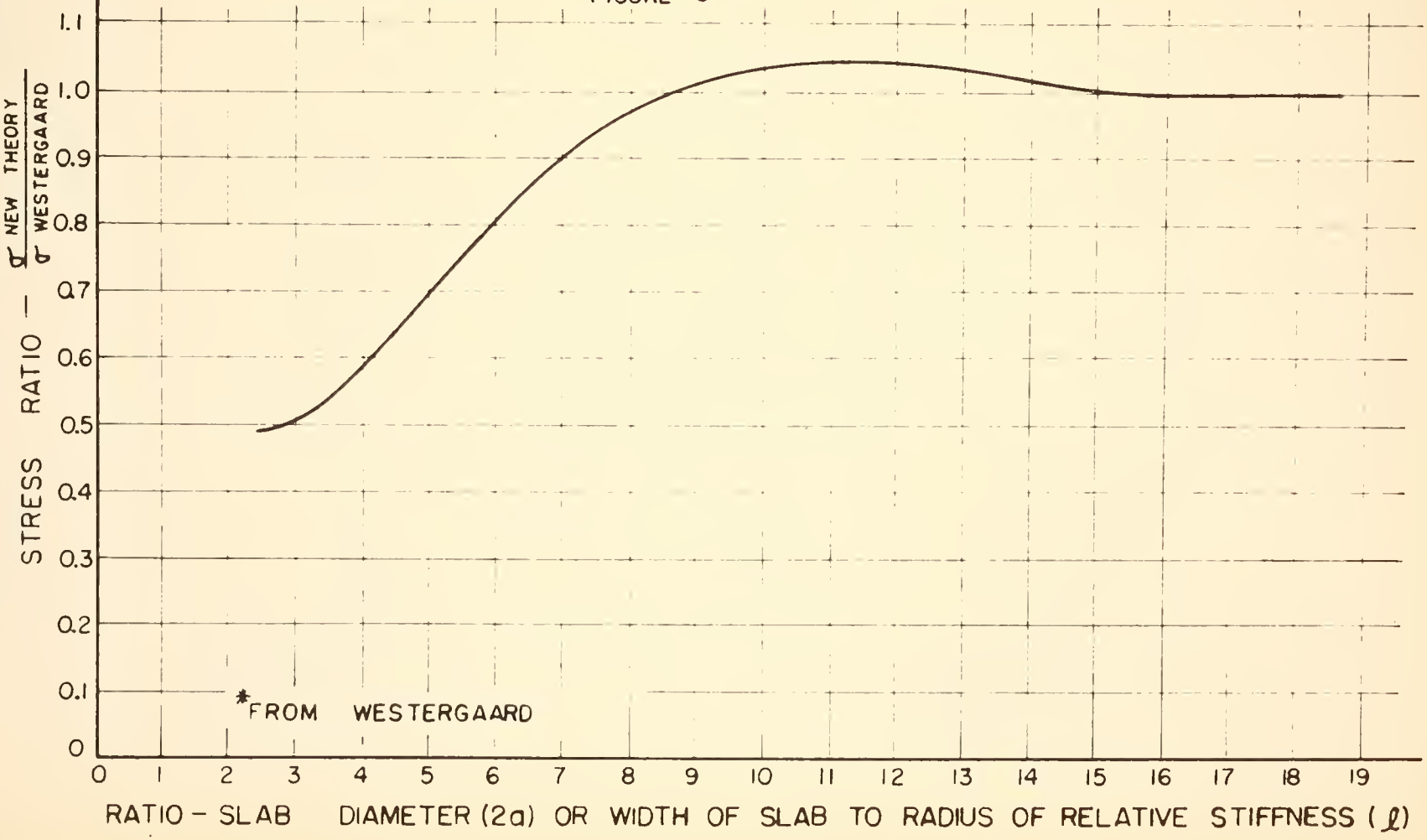
E = 3,000,000 psi

FIGURE 7



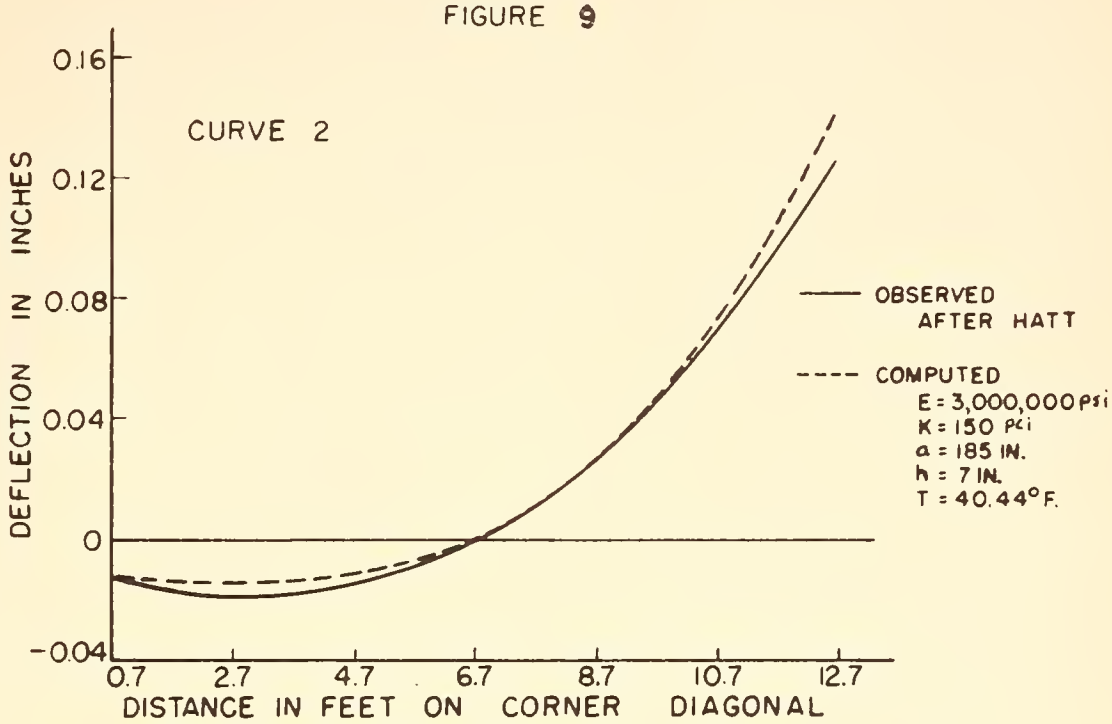
RATIO OF NORMAL STRESS AT CENTER OF CIRCULAR SLAB TO NORMAL STRESS AT CENTER OF INFINITELY LONG STRIP OF FINITE WIDTH* FOR CONDITIONS OF COMPLETE SUPPORT

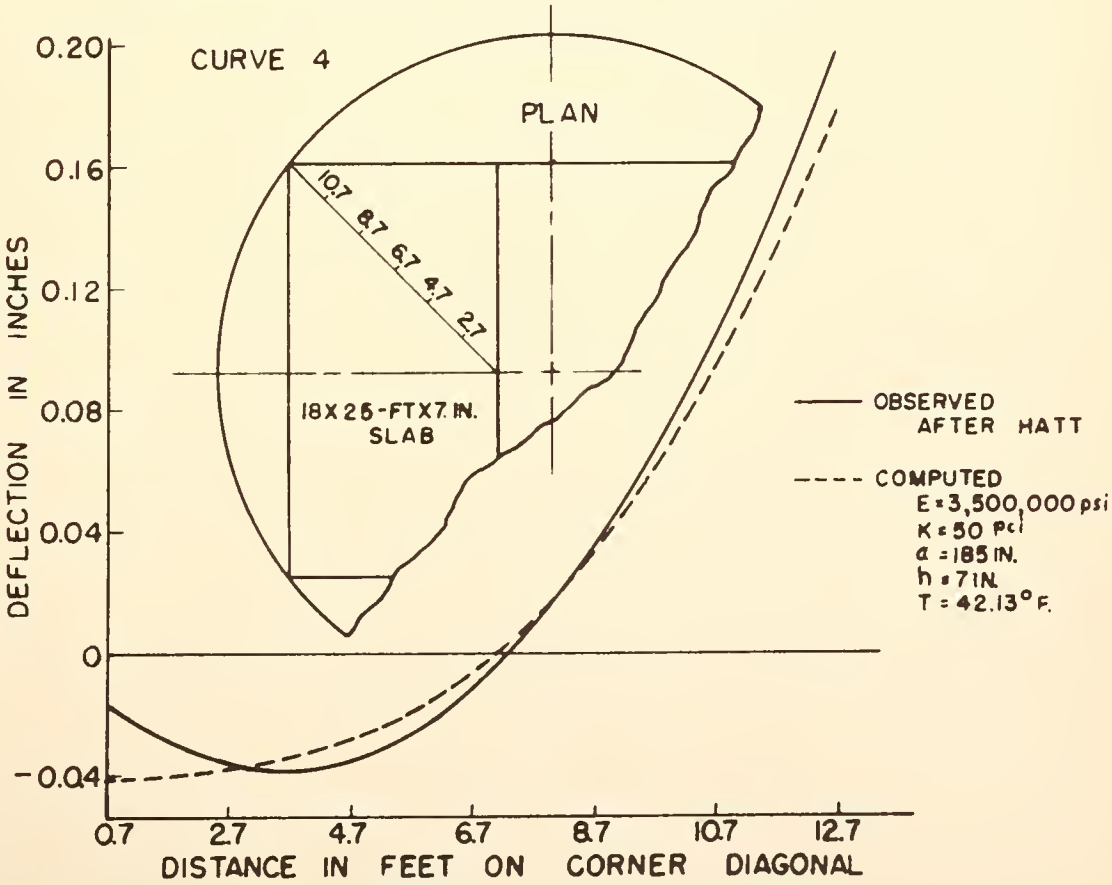
FIGURE 8



COMPARSION BETWEEN OBSERVED AND COMPUTED DEFLECTIONS — HATT

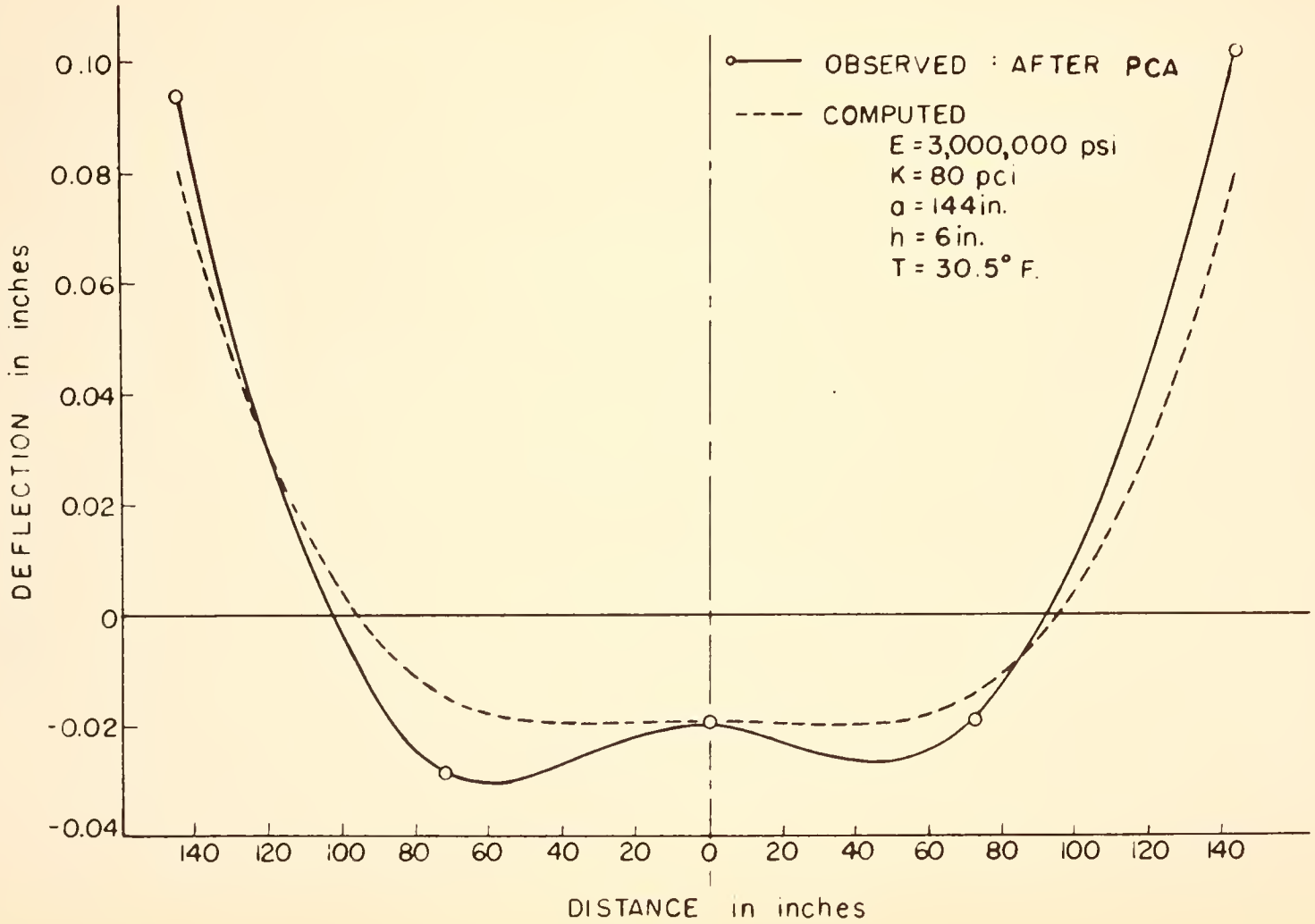
FIGURE 9





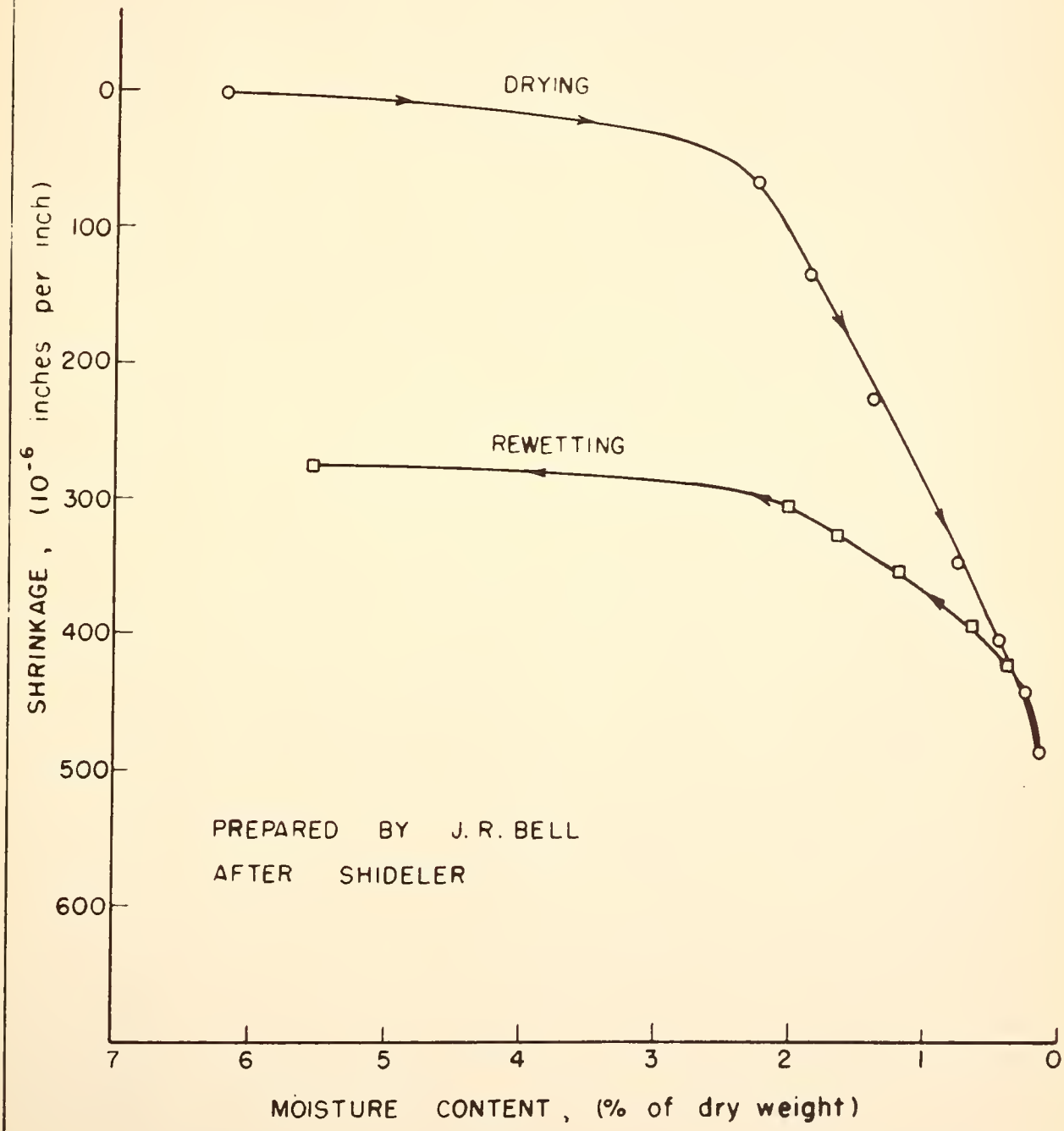
COMPARSION BETWEEN OBSERVED AND COMPUTED DEFLECTIONS — PORTLAND CEMENT ASSOCIATION

FIGURE 10



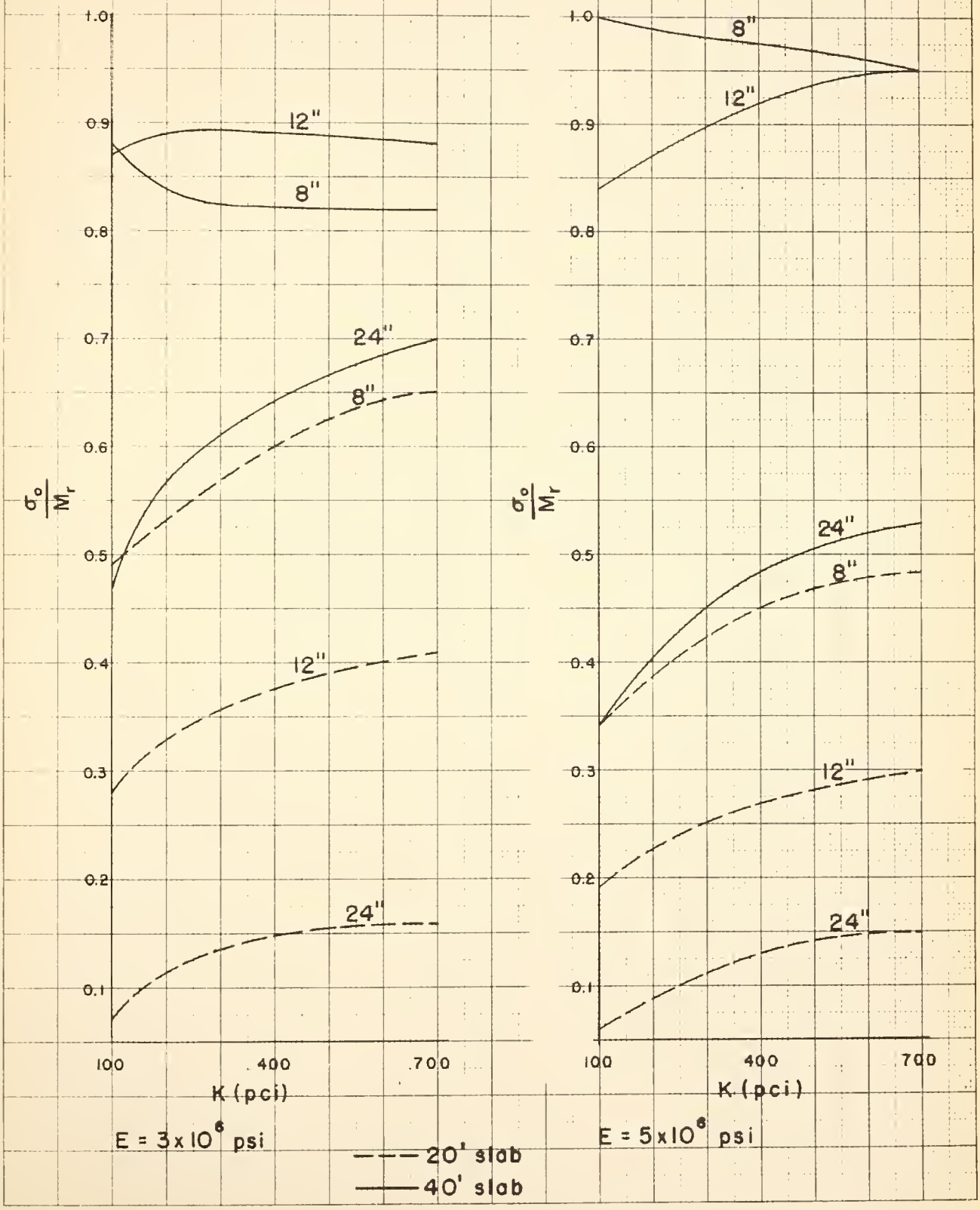
SHRINKAGE VS WATER CONTENT
FOR CONCRETE MASONRY UNITS

FIGURE 11



EQUAL EQUIVALENT TEMPERATURE DIFFERENCE (40°F)
BETWEEN TOP AND BOTTOM OF SLABS

FIGURE 12



EQUAL EQUIVALENT TEMPERATURE GRADIENTS ($\frac{5}{3}$ °F/inch)

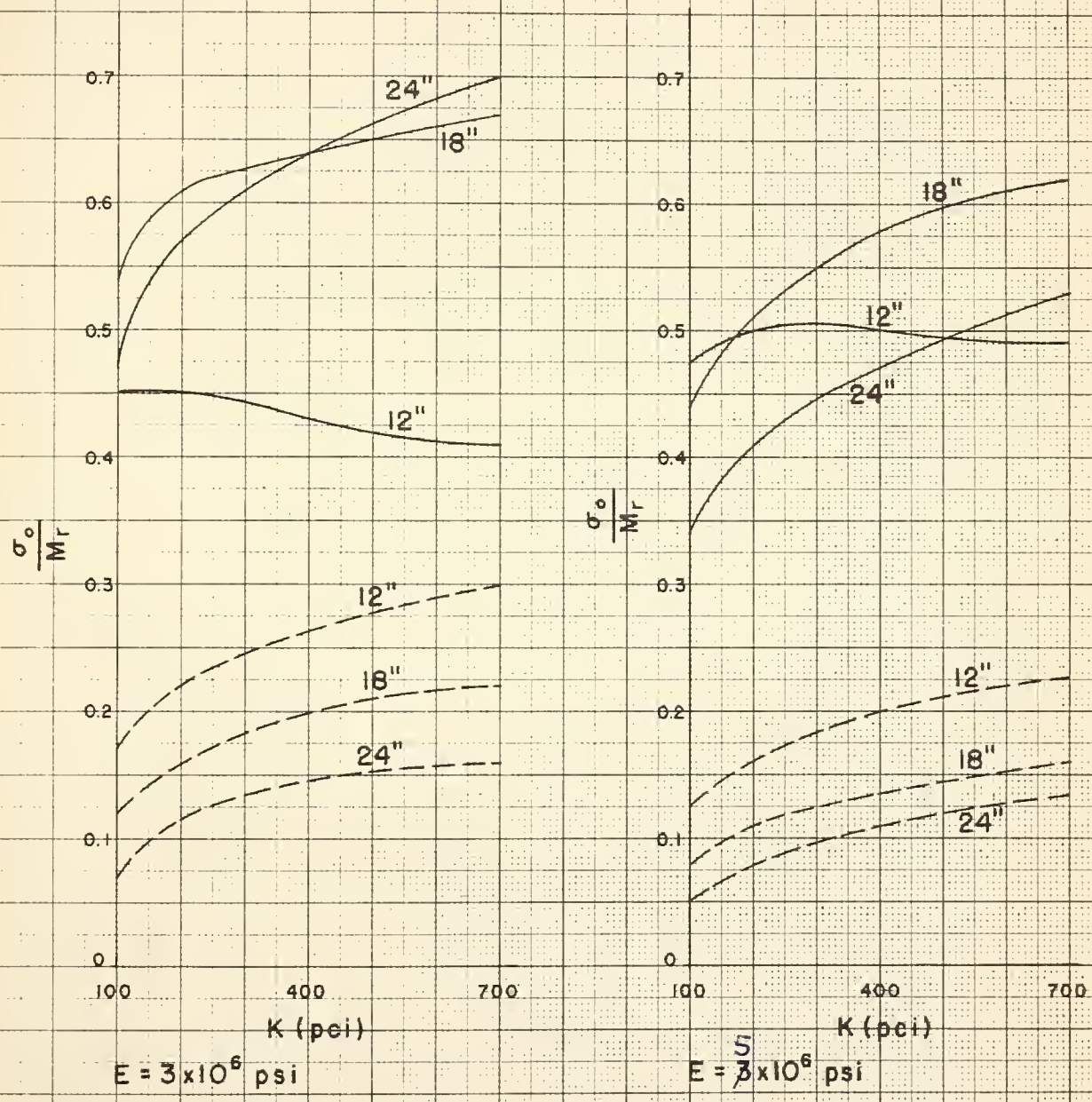



FIGURE 13